

Ambiguity Aversion in Competitive Insurance Markets: Adverse and Advantageous Selection

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Abstract:

We analyze an extension of the Rothschild-Stiglitz model with ambiguous loss probabilities and ambiguity-averse consumers to determine whether there are adverse or advantageous selection equilibria. We show that non-increasing absolute ambiguity aversion is sufficient for adverse selection. We show generally that actuarially fair pricing is also sufficient for adverse selection and that advantageous selection equilibria are inefficient. We then characterize the effect of ambiguity on the Rothschild-Stiglitz (RS) and Wilson-Miyazaki-Spence equilibrium under ambiguity aversion. In general, there is a deductible effect and an ambiguity effect both of which influence the critical proportion of high risks required for the RS equilibrium to exist and social welfare. We derive conditions under which ambiguity increases or decreases social welfare.

Keywords: adverse selection, ambiguity, efficiency, second best, welfare.

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1. Introduction

Self-selection based on policyholders' private information plays a central role in the performance of insurance markets. Standard models of asymmetric information (e.g., Rothschild and Stiglitz, 1976, Arnott and Stiglitz, 1988) predict a positive correlation between coverage and ex-post risk. Chiappori et al. (CJSS, 2006) show that this positive correlation is a robust prediction of the economic theory of asymmetric information. However, the empirical evidence on the correlation between coverage and risk in insurance markets is mixed.¹ Hemenway (1990, 1992) reports that individuals who engage in risky behavior (e.g., riding a motorcycle without a helmet) are less likely to have insurance. Chiappori and Salanié (2000) report no correlation among beginning French drivers, while Cohen (2005) reports a positive correlation for experienced Israeli drivers. Finkelstein and Poterba (2002, 2004) and He (2009) provide evidence of adverse selection in annuity markets and life insurance markets, respectively. Cawley and Phillipson (1999) report that mortality is lower for individuals with life insurance. Hendel and Lizzeri (2003) also report evidence of advantageous selection in the life insurance market. Cardon and Hendel (2001) find no evidence of asymmetric information in health insurance, while Bundorf, Herring and Pauly's (2010), Handel's (2013) and Bajari et. al.'s (2014) findings are consistent with adverse selection in employer provided health insurance.² Bolhaar, Lindebloom and van der Klaaw (2012) report

¹ See Cohen and Siegelman (2010) and Chiappori and Salanié (2013) for reviews of the literature on empirical analyses of asymmetric information.

² Cutler and Zeckhauser (2000), in a review of the earlier literature on health insurance, report that the vast majority of studies find evidence of adverse selection.

evidence of advantageous selection in the Irish supplemental health insurance market. Cutler, Finkelstein and McGarry (2008) report positive correlations for annuities and health insurance, negative correlations for life and Medigap insurance and no correlation for long term care insurance.

Recent theoretical research has attempted to explain this empirical evidence. Hemenway (1990), De Meza and Webb (2001) and De Donder and Hicks (2009) propose preference-based explanations. They argue that more risk averse individuals are more likely to take actions to reduce their risk exposure and also more likely to purchase insurance or to purchase more coverage. Netzer and Scheuer (2010) assume people differ with respect to productivity or patience in addition to risk. Individuals who are more productive or more patient accumulate more wealth, which reduces their marginal willingness to pay for insurance. They show that this implies there is no necessary correlation between coverage and risk. Huang, Liu and Tzeng (2010) analyze a model in which individuals may be overconfident. Individuals who underestimate their risk chose low effort, and individuals who do not underestimate their risk chose high effort. In equilibrium, insurers offer high coverage policies to attract the rational (low risk) types and low coverage contracts to attract the overconfident (high risk) types. Spinnewijn (2013) assumes that individuals have different perceptions of both their basic risk and their ability to affect the risk. Depending on the correlation of individuals' beliefs about the basic risk and the ability to control the risk, there may be a positive or negative correlation between coverage and ex-post risk. All of these models posit a second dimension of private information which, given the appropriate correlation with risk type, leads to a violation of the single-crossing condition and allows for the possibility of negative correlation between coverage and risk.

Virtually all of the theoretical analyses of adverse and advantageous selection assume that the relevant loss distributions are known or can be learned with certainty. However, even with the best available information there may be “uncertainty about probability created by missing information that is relevant and could be known” (Camerer and Weber 1992, p. 330). There are several reasons why decision-makers might experience uncertainty about their true accident probability. Anderson (2002) describes the ambiguity surrounding environmental risks, even for risk-neutral corporations. Knowledge of family health history may suggest an individual has above or below average propensity to develop a disease, but not the exact risk of becoming symptomatic. For new products, engineering studies will provide, at best, estimates of the risk of injury subject to some estimation error.

In this paper we develop a model of risk-based self-selection in which otherwise identical individuals differ only with respect to their loss probabilities. We assume that the loss probabilities are not completely knowable, so that there is ambiguity, and that individuals are ambiguity-averse. We employ the “smooth” model of ambiguity aversion (Klibanoff, Marinacci and Mukerji 2005, Neilson 2010).³ Our objective is to keep as close to the original Rothschild-Stiglitz (1976) framework as possible, so we make no further modifications to the model. In particular, we assume individuals face only an ambiguous risk which may be either high or low. We also assume that the degree of ambiguity and of ambiguity aversion is the same for all individuals. We analyze the existence and characteristics of equilibria in competitive insurance markets with asymmetric information when there is ambiguity regarding the loss probabilities and individuals are ambiguity-averse. Snow (2009) argues forcefully that competition in

³ The smooth model is not without controversy, see Epstein (2010) and Klibanoff, Marinacci and Mukerji (2012). Etner, Jaleva and Tallon (2012) provide a survey of models of decision-making under ambiguity.

insurance markets implies that firms earn zero expected profits in equilibrium. Consequently, we focus on zero expected profit equilibria.

Insurance markets with asymmetric information and ambiguity have also been examined by Koufopoulos and Kazhan (KK, 2014, 2015). They assume that policyholders have maxmin expected utility preference (e.g., Gilboa and Schmeidler, 1989) and allow both ambiguity and ambiguity aversion to vary between high and low risks. KK (2014) find that, depending on the range of possible loss probabilities for high and low risk there can be a pooling Rothschild-Stiglitz equilibrium and there may be a separating equilibrium. KK (2015) employ an idiosyncratic assumption about insurers' commitment to offer policies. That is, insurers may offer some policies that they can later withdraw from the market and other policies that they cannot withdraw. This commitment assumption implies that equilibrium always exists and is unique. All of the equilibria in their model are second-best efficient. In their model there can be a pooling equilibrium at full coverage for both types. There is also a separating equilibrium in which high risks fully insured and low risks over-insure.

Insurance markets with asymmetric information and ambiguity have also been examined by Huang, Snow and Tzeng (HST, 2015). Their model is the same as ours, although they interpret ambiguity aversion in terms of the failure to reduce compound lotteries. HST assume that individuals have "smooth" ambiguity aversion, which they interpret as reflecting attitudes toward compound lotteries. They provide two main results. First, they show that increasing absolute ambiguity aversion is sufficient for the single crossing property to fail. Second, they show that if the high and low risk indifference curves cross twice, then a separating equilibrium with advantageous selection may exist.

Our work differs from the above mentioned papers in several dimensions. We use the smooth ambiguity model for two main reasons. First, it allows the separation of ambiguity from attitudes toward ambiguity. Second, it contains the maxmin model as the special case where all of the weight is put on the worst possible loss probability. This allows us to generalize several of the results in KK's work. We show that non-increasing ambiguity aversion is sufficient for indifference curves to have the single-crossing property. This is essentially the converse of HST's result for increasing ambiguity aversion. We show that, under general assumptions and whether or not single-crossing holds, actuarially fair pricing sufficient to ensure that equilibria are characterized by adverse selection. We examine second-best efficiency under general assumptions, and show that the second-best contracts lead to adverse selection. That is, advantageous selection equilibria are inefficient. We then examine asymmetric information equilibria in the smooth model. We analyze the characteristics and existence of equilibrium in the Rothschild-Stiglitz under the assumption single-crossing does not hold. We then characterize Wilson (1977)-Miyazaki (1977)-Spence (1978, WMS) equilibrium under smooth ambiguity aversion. This is, at least to our knowledge, the first analysis of second-best efficiency or the WMS equilibrium under the smooth ambiguity model. We then return to the single-crossing case and address two questions. First, how does ambiguity affect the existence of the Rothschild-Stiglitz equilibrium? Second, how does the introduction of ambiguity affect welfare?

The second section briefly reviews that standard model without ambiguity. Section 3 enriches the standard model with ambiguity and ambiguity aversion and provides basic results. Section 4 analyzes the model under the assumption that single-crossing does not hold and advantageous selection equilibria may potentially arise. In Section 5 we analyze the effect of ambigui-

ty aversion on the existence of the RS equilibrium and on the consumer welfare. Section 6 provides brief concluding remarks.

2. The Insurance Market with Asymmetric Information

In this section we set out the basic model of insurance markets with asymmetric information in the absence of ambiguity. We assume that applicants for insurance are endowed with initial wealth W . Risk preferences are characterized by the increasing and concave vNM utility function $u(\cdot)$. The individuals in our model incur a loss l with probability π^H if they are high risk, or the lower probability π^L if they are low risk, $0 < \pi^L < \pi^H < 1$. The proportion of high risks in the population is given by λ . We denote by W_N wealth in the no-loss state and by W_A wealth in the accident state when the loss is incurred, implying that $W_N - W_A$ is the portion of the loss retained by the consumer (i.e., his deductible). All of these parameters of the model are common knowledge, but whether a given individual is high or low risk is private information. Expected utility is given by $U^t(W_N, W_A) = (1 - \pi^t)u(W_N) + \pi^t u(W_A)$ for a type t individual, $t \in \{H, L\}$. The indifference curves are downward sloping and convex in state space. The indifference curves satisfy the single-crossing condition – the low risk indifference curves are steeper than the high risk indifference curves at any point (W_N, W_A) . An insurance policy consists of a premium, p , paid by the insured in both states of the world and an indemnity, q , paid to the insured if a loss occurs so that $W_N = W - p$ and $W_A = W - p - l + q$. An insurance policy or contract can be identified as $C = (W_N, W_A)$, specifying wealth in the no-loss state and the loss state. The insurer's expected profit from a policy sold to a type t consumers is given by $\Pi^t = W - \pi^t l - [(1 - \pi^t)W_N + \pi^t W_A]$, $t \in \{H, L\}$.

In the Rothschild-Stiglitz model, firms simultaneously decide which one contract to offer. Consumers then simultaneously decide which policy to buy. Rothschild and Stiglitz (1976) define equilibrium as the set of contracts such that, when customers maximize expected utility, (i) no contract makes negative expected profits in equilibrium; and (ii) there is no contract outside the equilibrium set that, if offered, will generate a non-negative profit. The Rothschild-Stiglitz equilibrium is illustrated in Figure 1. The point E is the initial endowment, and the lines EL , EF and EH are the fair odds lines for the low risks, population average and high risks, respectively. The contracts must break even individually and satisfy self-selection. Competition for the high risks leads to fairly priced full coverage at the point H^{RS} with wealth $W^{H,RS} = W - \pi^H l$ in both states of the world. The binding self-selection constraint is then

$$U^H(W^{H,RS}) = (1 - \pi^H)u(W_N^L) + \pi^H u(W_A^L). \quad (2.1)$$

The low-risk contract provides fairly priced partial coverage at L^{RS} at the intersection of the high-risk indifference curve U^H and the low-risk fair odds line. We will refer to the contracts $H^{RS} = (W^{H,RS}, W^{H,RS})$ and $L^{RS} = (W_N^{L,RS}, W_A^{L,RS})$ as the Rothschild-Stiglitz (RS) contracts.

Whether the RS contracts are actually an equilibrium depends on whether the proportion of high risks is above the critical value λ^{RS} . If $\lambda \geq \lambda^{RS}$, then the RS contracts are an equilibrium. If $\lambda < \lambda^{RS}$, then the RS equilibrium does not exist. Suppose that the proportion of high risks is such that the population fair odds line is EF' in Figure 1. Then there is a pooling contract, such as the one at X in Figure 1, which attracts both types of consumers and earns a positive profit. As a result, the RS contracts cannot be sustained as an equilibrium. The pooled policy cannot be sustained as an equilibrium either, since the single-crossing property implies that the low risks can always be attracted away from a pooled policy that makes non-negative expected profits.

The assumption that firms can only offer one policy is an important restriction. Suppose instead, firms are allowed to offer a menu of contracts. This leads to the Wilson (1977)-Miyazaki (1977)-Spence (1978) (WMS) equilibrium. The equilibrium is the pair of policies that maximizes the expected utility of the low risks, subject to the self-selection constraint (2.1) and the zero expected profit condition $\lambda\Pi^H + (1 - \lambda)\Pi^L = 0$. The feasible contract curve (FCC) is the locus of low-risk policies the resource constraint and self-selection constraints as equalities for some high risk full insurance contract. If the proportion of high risks is large enough then the WMS equilibrium coincides with the RS equilibrium. The RS equilibrium can be broken by a firm offering a menu of contracts at a higher proportion of high risks compared to a situation where firms can only offer single contracts. The critical value, λ^{WMS} , is determined by the tangency of the FCC and the low risk indifference curve through L^{RS} . Since the FCC lies above the pooled fair odds line, the critical value for which the RS contracts are the WMS equilibrium, λ^{WMS} , is larger than λ^{RS} (Crocker and Snow, 1985).⁴ If the proportion of high risks is less than λ^{WMS} , then the equilibrium contracts involve cross-subsidization from the low risks to the high risks. A cross-subsidized WMS equilibrium is illustrated in Figure 1. The curve AL^{RS} is the FCC and the equilibrium is the pair of points H^{WMS} and L^{WMS} . The high risks obtain full insurance, $H^{WMS} = (W_N^{H,WMS}, W_A^{H,WMS})$. The low-risk policy is at L^{WMS} , the tangency of the low risk indifference curve with the locus AL^{RS} . The low-risk policy, $L^{WMS} = (W_N^{L,WMS}, W_A^{L,WMS})$, provides partial coverage. The WMS equilibrium always exists and is second-best efficient by construction.

⁴ Thus, if firms can only offer one contract, then for $\lambda^{RS} \leq \lambda < \lambda^{WMS}$, the RS equilibrium exists but is not second best. This occurs if the pooled fair odds line lies below the low risk indifference curve through C^{L0} , but the ECL intersects the indifference curve.

3. Ambiguity Aversion

To incorporate ambiguity, we assume that the probability of loss is subject to uncertainty. It is given by $\tilde{\pi} = \pi + \tilde{e}$, where $\tilde{e}$ is a random variable with distribution F on the support $[\underline{e}, \bar{e}]$. We also assume that beliefs about the loss probability are unbiased, i.e. $E\{\tilde{\pi}\} = \pi$. Since consumers can observe the prices of the policies offered on the market, they can infer the expected probabilities of loss (Ligon and Thistle, 1996).⁵

There are several approaches to model ambiguity aversion. We employ the model of smooth ambiguity preferences developed by Klibanoff, Marinacci, and Mukerji (2005) and Neilson (2010). In this model individuals form an expectation of Φ -weighted expected utilities according to the second-order beliefs F . Φ' is positive because higher expected utility is desirable for the individual. The curvature of Φ captures ambiguity attitude: If Φ'' is negative (zero, positive), the decision-maker is ambiguity-averse (-neutral, -loving). In the case of ambiguity neutrality, our model collapses to the standard (subjective) EUT-case in which indifference curves are decreasing, convex, and satisfy the single-crossing property.

We focus on the case of ambiguity aversion, which is obtained for $\Phi'' < 0$. We denote by $\tilde{U} = \tilde{\pi}u(W_A) + (1 - \tilde{\pi})u(W_N)$ expected utility under ambiguous beliefs about the true probability of loss and by

$$V(W_N, W_A) = E\{\Phi(\tilde{U})\} = E\{\Phi(\tilde{\pi}u(W_A) + (1 - \tilde{\pi})u(W_N))\} \quad (3.1)$$

the objective function of an ambiguity-averse decision-maker. First, observe that the indifference curves are downward sloping. In order for V to stay constant, an increase in wealth in one state of the world must be offset by a decrease in wealth in the other state of the world. The slope of the indifference curve is given by

⁵ To ensure proper beliefs throughout the analysis we assume $\underline{e} > -\pi^L$ and $\bar{e} < 1 - \pi^H$.

$$M(\pi) = \left. \frac{dW_A}{dW_N} \right|_{V=\text{const}} = - \frac{E\{\Phi'(\tilde{U})(1-\pi-\tilde{\epsilon})\}u'(W_N)}{E\{\Phi'(\tilde{U})(\pi+\tilde{\epsilon})\}u'(W_A)} < 0. \quad (3.2)$$

Convexity of the indifference curves in the state space follows from the concavity of u and Φ , which imply that V is a concave function of (W_N, W_A) . To see this, choose two points (W'_N, W'_A) and (W''_N, W''_A) on the same indifference curve so that $V(W'_N, W'_A) = V(W''_N, W''_A)$. Then,

$$V(sW'_N + (1-s)W''_N, sW'_A + (1-s)W''_A) > sV(W'_N, W'_A) + (1-s)V(W''_N, W''_A)$$

for all $0 < s < 1$. The point $(sW'_N + (1-s)W''_N, sW'_A + (1-s)W''_A)$ lies on a higher indifference curve and it follows that the indifference curves are convex.⁶

Observe that at full insurance ($W_N = W_A$), the slope of the indifference curve is given by $-(1-\pi)/\pi$. As a result the indifference curve is tangent to the fair odds line at full insurance. This implies that individuals fully insure when prices are actuarially fair. Alternatively, write the insurance premium as $p = (1+\gamma)\pi q$, and substitute it into V to obtain

$$V = E\{\Phi(\tilde{\pi}u(W - (1+\gamma)\pi q - l + q) + (1-\tilde{\pi})u(W - (1+\gamma)\pi q))\}.$$

Differentiating with respect to q yields the following first-order expression:

$$dV/dq = E\{\Phi'(\tilde{U})[(1 - (1+\gamma)\pi)\tilde{\pi}u'(W_A) - (1+\gamma)\pi(1-\tilde{\pi})u'(W_N)]\}.$$

If insurance is actuarially fair ($\gamma = 0$), this expression is equal to zero at $W_N = W_A$. For $\gamma > 0$, the derivative is negative when evaluated at $W_N = W_A$. Thus, for individuals who are ambiguity-averse, Mossin's (1968) Theorem holds: individuals buy full insurance at fair prices and less than full insurance at unfair prices. This result is consistent with Alary et al. (2013).

We now turn to the effects of ambiguity and ambiguity aversion on the shape of the indifference curves. Following Pratt (1964), one agent is more risk-averse than another if he

⁶ If the individual is ambiguity loving, the indifference curves need not be convex. We do not consider this case.

dislikes every lottery that the other agent dislikes. In our model, ambiguity aversion makes the individual more risk-averse to lotteries over wealth. From the concavity of Φ we infer that

$$\begin{aligned} V(W_N, W_A) &= E\{\Phi(\tilde{\pi}u(W_A) + (1 - \tilde{\pi})u(W_N))\} \leq \Phi(E\{\tilde{\pi}u(W_A) + (1 - \tilde{\pi})u(W_N)\}) \\ &= \Phi(\pi u(W_A) + (1 - \pi)u(W_N)) = \Phi(U(W_A, W_L)). \end{aligned}$$

Since $\Phi(U)$ and U rank lotteries over wealth the same, the ambiguity-averse agent dislikes every lottery the ambiguity-neutral one dislikes. With a similar argument, an increase in ambiguity aversion, represented by an increasing and concave transformation of Φ , and an increase in ambiguity, represented by a mean-preserving spread in second-order beliefs F , result in stronger risk aversion towards lotteries over wealth.⁷ As a consequence, indifference curves at less than full insurance ($W_N > W_A$) are flatter in the presence of ambiguity than in its absence because their slopes are less negative at a given state-contingent wealth pair.⁸

Before turning to the equilibrium analysis, we investigate how single-crossing is affected by the presence of ambiguity. The single-crossing property holds if the low-risk indifference curves are always steeper than the high-risk indifference curves, $M(\pi^L) < M(\pi^H)$. At full insurance we have $M(\pi^L) = -(1 - \pi^L)/\pi^L < -(1 - \pi^H)/\pi^H = M(\pi^H)$, so that the single-crossing property is satisfied. In general, $M(\pi^L) < M(\pi^H)$ is equivalent to:

$$\begin{aligned} (\pi^H - \pi^L) E\{\Phi'(\tilde{U}^H)\}E\{\Phi'(\tilde{U}^L)\} + E\{\Phi'(\tilde{U}^H)\tilde{e}\}E\{\Phi'(\tilde{U}^L)\} \\ - E\{\Phi'(\tilde{U}^H)\}E\{\Phi'(\tilde{U}^L)\tilde{e}\} > 0 \end{aligned} \quad (3.3)$$

The first term is positive. Since $E\{\Phi'(\tilde{U}^t)\tilde{e}\}$ is positive for both $t \in \{H, L\}$, whether the inequality in (3.3) holds depends on the relative magnitudes of the second and third term. Also notice

⁷ It is well known that an increase in risk aversion decreases the demand for the risky asset (Pratt, 1964). An increase in ambiguity aversion, however, need not decrease the demand for an ambiguous asset (Gollier, 2011). In our case, the comparative statics are clear because we use a binary loss distribution.

⁸ This increased aversion to lotteries over wealth underlies the finding that ambiguity aversion increases the willingness to pay for insurance (see Alary, Gollier and Treich, 2013; Bajtelsmit, Coats and Thistle, 2015; Snow, 2011).

that the ambiguity experienced by the high (low) risks makes the single-crossing property more (less) likely to be satisfied. This shows that ambiguity has different implications depending on which risk type experiences it despite the fact that both high and low risks face the same level of ambiguity. To resolve this indeterminacy, we utilize the index of absolute ambiguity aversion, $A_\Phi(U) = -\Phi''(U)/\Phi'(U)$ and speak of increasing (constant, decreasing) ambiguity aversion if $A_\Phi(U)$ is an increasing (constant, decreasing) function of utility.⁹ The following proposition presents a simple sufficient condition for the single-crossing property to be globally satisfied.

Proposition 1: Non-increasing ambiguity aversion is sufficient for the single-crossing property to hold. Increasing ambiguity aversion is necessary but not sufficient for the single-crossing property to fail.

Proof: See Appendix 1.

This condition is simple and intuitive. Our analysis above shows that it is the ambiguity experienced by the low risks that might lead to the failure of the single-crossing property. For a given state-contingent wealth profile, low risks have higher expected utility than high risks because they are less likely to be in the low-wealth state. Under non-increasing ambiguity aversion, higher levels of expected utility imply that a given level of ambiguity is less painful than at lower levels of expected utility (or at least not more painful). Therefore, although high risks and low risks experience the same level of ambiguity, low risks are affected less as soon as ambiguity aversion is non-increasing. As such, their propensity to exchange wealth in the no-loss state for wealth in the accident state increases by less than that of the high risks, which guarantees single-crossing.¹⁰

⁹ Cherbonnier and Gollier (2015) show that if both u and Φ exhibit decreasing concavity, this ensures that the agent is “decreasingly averse” so that a reduction in wealth can never make an undesirable uncertain prospect desirable.

¹⁰ In Appendix 2 we derive the sufficient condition for Proposition 1 to hold when ambiguity takes the multiplicative form $\tilde{\pi} = (1 + \tilde{\epsilon})\pi$.

Conversely, a necessary condition when single-crossing is violated, is that absolute ambiguity aversion is increasing (see also Proposition 1 in HST, 2017). If single-crossing fails, then the low-risk indifference curve crosses the high-risk indifference curve from above at full insurance and then crosses the high-risk indifference curve again from below. This makes advantageous selection equilibria possible.

4. Advantageous Selection in Competitive Insurance Markets

In this section we analyze whether equilibria with advantageous selection can exist in competitive insurance markets when prices are actuarially fair. We first provide two results that hold under general assumptions and are not specific to smooth ambiguity aversion. We show that fair pricing implies that high risks obtain full coverage and that the correlation between coverage and ex-post risk is non-negative. We derive the second-best efficient contracts and show that advantageous selection equilibria are inefficient. We then turn to results that are specific to smooth ambiguity aversion. We provide a detailed characterization of the RS equilibrium, including conditions for the equilibrium to exist. We characterize the WMS equilibrium and compare the equilibria with and without ambiguity.

4.1 Fair Pricing and Efficiency. In this subsection, we assume preferences are $\mathcal{V}^i(C) = \mathcal{V}^i(W_N, W_A)$, $i = H, L$ where the $\mathcal{V}^i$ are strictly increasing and concave, continuous and satisfy Mossin's theorem. We assume that contracts must be resource feasible, so that $\lambda \Pi^H(C^H) + (1 - \lambda) \Pi^L(C^L) \geq 0$, where C^H and C^L are the contracts bought by the high and low risks. Any transfers to or from policyholders must be lump sum transfers, and net transfers must be non-positive. We assume that the contracts satisfy the self-selection constraints $\mathcal{V}^H(C^H) \geq \mathcal{V}^H(C^L)$ and $\mathcal{V}^L(C^L) \geq \mathcal{V}^L(C^H)$. We assume that (C^H, C^L) cannot be an equilibrium outcome if

there is another resource- and incentive-feasible contract pair that makes both types better off and/or increases expected profit. These conditions are part of any reasonable equilibrium concept in insurance markets with asymmetric information. We assume that firms are risk- and ambiguity-neutral. Finally, we assume that prices are actuarially fair at the margin, so that $dW_A^t = \left(\frac{1-\pi^t}{\pi^t}\right) dW_N^t$, for $t \in \{H, L\}$. This allows for two-part pricing (Oi, 1971), taxes and cross-subsidies. We make no assumption about the crossing of the indifference curves.

Proposition 2: Under the assumptions of the preceding paragraph, in equilibrium (a) the high risks receive full coverage and (b) the correlation between coverage and ex-post risk is non-negative.

Proof: (a) Let (C^H, C^L) be proposed equilibrium contracts, where $C^H = (W_N^H, W_A^H)$ offers less than full insurance, $W_N^H > W_A^H$. Let $C^{H'} = (W_N^H - \delta_1, W_A^H + \delta_2)$ where $\delta_2 = \frac{1-p^H}{p^H} \delta_1, \delta_1 > 0$. Let $C^{L'} = (W_N^L - \varepsilon_1, W_A^H + \varepsilon_2)$ where $\varepsilon_2 = \frac{1-p^L}{p^L} \varepsilon_1$. Observe that $\mathcal{V}^i(C^{j'}) > \mathcal{V}^i(C^j), i, j = H, L$.

If both self-selection constraints are slack at (C^H, C^L) , then the welfare of both types can be increased and/or profits increased. This cannot be an equilibrium, so at least one self-selection constraint must be binding. Suppose that both self-selection constraints are binding. This must be at a pooled policy, $C^H = C^L$. But both types can be made better off and/or profits increased at $C^{H'}$. If there is a pooling equilibrium, it must be at full insurance. Now suppose that the high risk self-selection constraint is slack and the low risk self-selection constraint is binding, $\mathcal{V}^L(C^L) = \mathcal{V}^L(C^H)$. Again, both types can be made better off and/or profits increased at $C^{H'}$. Finally, suppose that the low risk self-selection constraint is slack and the high risk self-selection constraint is binding, $\mathcal{V}^H(C^H) = \mathcal{V}^H(C^L)$. Then $\mathcal{V}^H(C^{H'}) > \mathcal{V}^H(C^L)$. There is a $C^{L'}$ such that $\mathcal{V}^H(C^{H'}) = \mathcal{V}^H(C^{L'})$. Both types can be made better off and/or profits increased at $(C^{H'}, C^{L'})$. If C^H offers more than full insurance, take $\delta_1 < 0$. If C^H does not offer full insurance, then

(C^H, C^L) cannot be an equilibrium. (b) Since the high risk obtain full coverage, a negative correlation between coverage and ex-post risk implies the low risks overinsure. But if prices are actuarially fair, the unconstrained optimum for the low risks is full coverage, in which case the correlation is zero. If the self-selection constraint for the high risks is binding, the expected payoff for the low risks cannot increase. This implies that the coverage of the low risks cannot increase. This implies that the correlation between coverage and ex-post risk is non-negative. ||

This result is fairly general. It does not assume any particular model of preferences, only that preferences are averse to lotteries over wealth. It includes the expected utility model, the maxmin expected utility model, the α -maxmin utility model and the smooth model.¹¹ The result applies whether single-crossing holds or not. The result does not assume a particular equilibrium concept, rather, it relies on maximizing behavior by consumers and firms. The resource constraint need not bind. Thus, the result applies to competitive markets and to a monopolist using two-part prices. The key assumption is actuarially fair pricing. In particular, for advantageous selection to arise, prices must be actuarially unfair.

CJSS provide an important result showing that the positive correlation property holds under very general conditions. They assume that preferences are monotonic and risk averse to lotteries over wealth and that consumers have unbiased expectations ($E\{\tilde{\pi}\} = \pi$). Their result does not rely on any particular equilibrium concept. We make the similar assumptions. The key assumption in CJSS is “non-increasing profits”, that is, expected profits are weakly decreasing in the level of coverage. Since the policy that offers more coverage must sell for a higher price, this assumption implies that price cannot rise faster than the level of coverage. They show that these

¹¹ The α -maxmin utility model is introduced in Ghirardato, Maccheroni and Marinacci (2004) Let the loss probability π lie in $[\underline{\pi}, \bar{\pi}]$, and let $U(C|\pi)$ be expected utility with loss probability π . The α -maxmin expected utility is $V_\alpha(C) = \alpha U(C|\underline{\pi}) + (1 - \alpha)U(C|\bar{\pi})$. The parameter α is interpreted as a pessimism parameter, where $\alpha = 1$ yields the maxmin expected utility model. If the underlying vNM utility function is increasing and concave, then V_α is a convex combination of increasing, concave functions and is increasing and concave.

assumptions imply that the correlation between coverage and ex-post risk is non-negative. We replace the assumption of non-increasing profits with the assumption of actuarially fair pricing at the margin.¹²

We now turn to the analysis of the second best efficient contracts. The efficient contracts are the solution to the constrained maximization problem:

$$\max_{W_N^L, W_A^L} \mathcal{V}^L(W_N^L, W_A^L) \quad (4.1)$$

subject to

$$\mathcal{V}^H(W_N^H, W_A^H) \geq \mathcal{V}^H(W_N^L, W_A^L) \quad (4.2)$$

$$\mathcal{V}^L(W_N^L, W_A^L) \geq \mathcal{V}^L(W_N^H, W_A^H) \quad (4.3)$$

$$W - \bar{\pi}l - \lambda[\pi^H W_A^H + (1 - \pi^H)W_N^H] \quad (4.4)$$

$$-(1 - \lambda)[\pi^L W_A^L + (1 - \pi^L)W_N^L] \geq 0$$

$$\mathcal{V}^H(W_N^H, W_A^H) \geq \bar{\mathcal{V}}^H \quad (4.5)$$

Here (4.2) and (4.3) are the high-risk and low-risk self-selection constraints. Equation (4.4) is the resource constraint and (4.5) is the high-risk utility constraint. We let $\mathcal{V}_j^t(\cdot, \cdot), j \in \{N, A\}, t \in \{H, L\}$ denote the partial derivatives.

Proposition 3: Assume individuals are non-satiated, strictly averse to lotteries over wealth and have unbiased expectations; single-crossing need not hold. Let μ^H and δ be the Lagrangian multipliers for the high-risk self-selection constraint (4.2) and the high-risk utility constraint (4.5). The efficient contracts satisfy the following conditions:

- (a) The resource constraint is binding.
 - (b) The high risks are fully insured, $W_N^H = W_A^H = W^H$.
 - (c) The high-risk selection constraint is binding, $\mathcal{V}^H(W_N^H, W_A^H) = \mathcal{V}^H(W_N^L, W_A^L)$.
- If $\mathcal{V}^H$ and $\mathcal{V}^L$ are differentiable, then

¹² CJSS do not require actuarially fair pricing, only that prices increase more slowly than coverage. We assume marginal prices are actuarially fair but do not require profits weakly decrease with coverage. For example, in our analysis a monopolist could charge a higher fixed fee and earn more profit from high risks than low risks, violating NIP, so long as both types are charged actuarially fair marginal prices.

(d) The slope of the feasible contract curve is

$$\frac{dW_A^L}{dW_N^L} = - \frac{\lambda v_N^H(W_N^L, W_A^L) + (1-\lambda)v_N^H(W^H, W^H)(1 + \frac{\delta}{\mu^H})}{\lambda v_A^H(W_N^L, W_A^L) + (1-\lambda)v_N^H(W^H, W^H)(1 + \frac{\delta}{\mu^H})} \quad (4.6)$$

(e) The solution is unique.

The proof is given in Appendix 3. Again, this result does not assume any particular model of preferences. This is similar to the characterization of efficiency under expected utility (e.g., Crocker and Snow, 1986) and shows their analysis is actually quite general. Conditions (a), (b) and (c) are the same and condition (d) is the analog of the corresponding expression in Crocker and Snow. Uniqueness under expected utility is proved in Dionne and Fombaron (1996) and Netzer and Scheuer (2014); our proof is simpler and more general. We should point out that, since the high risks fully insure and the low risks partially insure (except at C^P), efficiency implies the correlation between coverage and ex-post risk is positive. Put differently, advantageous selection equilibria are inefficient.

4.2 Equilibria in the Smooth Model. We now examine equilibrium outcomes under the smooth model of ambiguity aversion. We explicitly assume the indifference curves cross twice and provide a more detailed analysis of the RS equilibrium. We compare the WMS equilibria with and without ambiguity.

Double crossing has important implication for the characterization and existence of equilibrium. Recall that the low risk indifference curve crosses the high risk indifference curve from above at full insurance then crosses the high risk indifference curve again from below. This implies that, given a high risk indifference curve, there is a point of tangency between the high risk indifference curve and a low risk indifference curve. Let C^{H0} be the fairly priced full insurance contract for the high risks. Let C^{L*} denote fairly priced low risk contract that satisfies the self-selection constraint constraints $V^H(C^{H0}) = V^H(C^{L*})$. Under single crossing (C^{H0}, C^{H*}) are the RS equilibrium contracts, this may not be the case under double crossing. Let C^{L**} be the point

of tangency between the high risk indifference curve through C^{H0} and a low risk indifference curve. Expanding our previous notation slightly, let $M(C, \pi^t)$ denote the type t marginal rate of substitution at the contract C .

Proposition 4: Assume that single-crossing does not hold. Assume that each firm can offer one policy and that the RS equilibrium exists. Then (a) if $M(C^{L*}, \pi^L) \leq M(C^{L*}, \pi^H)$, then (C^{H0}, C^{L*}) is the RS equilibrium and (b) if $M(C^{L*}, \pi^L) > M(C^{L*}, \pi^H)$, then (C^{H0}, C^{L**}) is the RS equilibrium.

Proof: (a) In this case the low risk indifference curve is steeper than the high risk indifference curve at C^{L*} , and it is “as if” single-crossing holds. (b) This is illustrated in Figure 2. The line LL is the low risk fair odds line. If $M(C^{L*}, \pi^L) > M(C^{L*}, \pi^H)$, then the low risk indifference curve (labeled $V^{L'}$) lies below the high risk indifference curve at C^{L*} (labeled V^H). Then there are policies below V^H and above $V^{L'}$ that attract low risks but not high risks and earn a positive profit. Then (C^{H0}, C^{H*}) cannot be an equilibrium. The policy C^{L**} is the policy most preferred by low risks that satisfies the self-selection constraint. Since the low risk indifference curve lies above the high risk indifference curve, any policy that attracts low risks also attracts high risks and is unprofitable. Therefore, (C^{H0}, C^{L**}) is the equilibrium. ||

If the low risk indifference curve through C^{L*} is steeper than the high risk indifference curve the resulting equilibrium is the standard RS equilibrium. In particular, firms earn zero expected profit in equilibrium. If the low risk indifference curve is flatter than the high risk indifference curve, there is still a separating equilibrium but it is one in which firms earn positive expected profits in equilibrium. The conclusion of positive expected profits rests critically on the assumption that firms can offer only one contract. A straightforward adaptation of the argument in Snow (2009) shows that the positive expected profit equilibrium can be broken if firms can offer menus of contracts.

Proposition 4 assumes that the equilibrium exists. The existence of equilibrium requires that the proportion of high risks be large enough that the pooled fair odds line lies below that low risk indifference curve through C^{L*} in case (a) or below the low risk indifference curve through C^{L**} in case (b). The flatter the relevant low risk indifference curve, the higher the proportion of high risk needed to support the existence of equilibrium. If the pooled fair odds line cuts the low risk indifference curve, then there is a pooled policy that breaks the separating equilibrium. But a pooled policy cannot be an equilibrium and the RS equilibrium fails to exist. This would seem to be a particular problem in case (b) where the low risk indifference curve is flatter than the high risk indifference curve at C^{L*} .

The FCC is the locus of low risk contracts that satisfy the resource and self-selection constraints. Let $C^P = (W^P, W^P)$ the population pooled full insurance contract at the intersection of the pooled fair odds line and the full insurance line. The FCC runs from the pooled full insurance contract, C^P , to the low risk RS equilibrium contract, C^{L*} . The slope of the FCC at C^P is $-(1 - \bar{p})/\bar{p}$. The FCC cannot be concave everywhere along its length, but there are no other apparent restrictions on the curvature of the feasible contract curve.¹³

The presence of ambiguity shifts the FCC.

Proposition 5: The feasible contract curve with ambiguity lies above the feasible contract curve without ambiguity.

Proof: The feasible contract curves coincide at C^P . Since the high risk indifference curves are flatter under ambiguity, the low risks receive more coverage at C^{L*} than at C^{L0} , so C^{L*} lies above C^{L0} on the low risk fair odds line. Now suppose that the feasible contract loci intersect at some contract $\hat{C}^L = (\hat{W}_N^L, \hat{W}_A^L)$. Then there is a high risk contract $\hat{C}^H = (\hat{W}^H, \hat{W}^H)$ that satisfies the

¹³ See Dionne and Fombaron (1996), who show that, under expected utility, the FCC may be convex, but cannot be concave everywhere along its length.

self-selection constraint with ambiguity, $V^H(\widehat{W}^H, \widehat{W}^H) = \Phi(u(\widehat{W}^H)) = V^H(\widehat{W}_N^L, \widehat{W}_A^L)$. There is also a high risk contract $\bar{C}^H = (\bar{W}^H, \bar{W}^H)$ that satisfies the self-selection constraint without ambiguity, $u(\bar{W}^H) = (1 - p^H)u(\widehat{W}_N^L) + p^H u(\widehat{W}_A^L)$. Then

$$\begin{aligned}\Phi(u(\bar{W}^H)) &= \Phi((1 - p^H)u(\widehat{W}_N^L) + p^H u(\widehat{W}_A^L)) \\ &> E\left\{\Phi\left((1 - \tilde{p}^H)u(\widehat{W}_N^L) + \tilde{p}^H u(\widehat{W}_A^L)\right)\right\} = \Phi(u(\widehat{W}^H)).\end{aligned}\quad (4.7)$$

This implies $\bar{W}^H > \widehat{W}^H$. If the contracts $(\hat{C}^H, \hat{C}^L)$ satisfy the resource constraint, then the contracts $(\bar{C}^H, \hat{C}^L)$ violate the resource constraint. ||

If we choose some level of wealth for the high risks, W^H , between W^{H*} and W^P , this determines a high risk utility constraint, $\Phi(u(W^H)) = \bar{V}^H$. Together with the resource constraint and the self-selection constraint, this determines the efficient low risk contract without ambiguity, say, $\bar{C}^L$, and the efficient low risk contract with ambiguity, say, $\hat{C}^L$. The subsidy to the high risks is the same in both cases. Then the tax on the low risks must also be the same in both cases. The contract $\hat{C}^L$ lies above $\bar{C}^L$ on the same low risk iso-profit contour. Then the low risks have more coverage under $\hat{C}^L$ than under $\bar{C}^L$; the additional coverage compensates for the ambiguity.

The WMS equilibrium is the solution to the second best efficiency problem with the high risk utility constraint set at $\bar{V}^H = V^H(W^{H0}, W^{H0})$; the constraint may or may not be binding. If the constraint is binding, then the outcome is the RS equilibrium. If the constraint is not binding, then the low risks subsidize the high risks in equilibrium. This is true whether or not there is ambiguity.

While we know that ambiguity shifts the FCC up, this does not necessarily imply that ambiguity increases or decreases coverage. This is illustrated in Figure 3, which compares cross-subsidized equilibria, one without ambiguity and two with ambiguity. The point E is the endow-

ment and the lines EF and EL are the pooled and low risk fair odds lines. The line AA is the full insurance line. The population pooled policy is C^P . With no ambiguity, the RS equilibrium is C^{L0} , and the solid curve is the FCC. The WMS equilibrium is the point WMS^0 , where the low risk indifference curve U^{L0} is tangent to the FCC. Under the first scenario with ambiguity, shown in Panel A, the RS equilibrium is C^{L1} and the FCC is the dashed curve from C^P to C^{L1} . The WMS equilibrium is at WMS^1 , the tangency of the low risk indifference curve V^{L1} , shown as a dashed curve, with the FCC. Comparing WMS^0 and WMS^1 , ambiguity leads to increased coverage for the low risks. Now consider a second scenario with ambiguity, shown in Panel B. In this scenario, the RS equilibrium is C^{L2} , and the FCC is the dotted curve from C^P to C^{L2} . The WMS equilibrium, WMS^2 , is at the tangency of the FCC and the indifference curve V^{L2} , shown as a dotted curve. Comparing WMS^0 and WMS^2 , ambiguity leads to decreased coverage for the low risks.

The figure is drawn under the assumption that the proportion of high risks is low enough that all three scenarios involve cross-subsidized equilibria. But ambiguity can change the critical proportion of high risks need to sustain a RS equilibrium. Then ambiguity may cause a shift from RS equilibrium to a cross-subsidized equilibrium, or *vice versa*.

5. The Rothschild and Stiglitz Equilibrium under Ambiguity Aversion

We now examine the effects of ambiguity on the existence of the RS equilibrium and on welfare. Throughout this section we assume that single-crossing is satisfied. Ambiguity raises the aversion towards lotteries over wealth. As a result, indifference curves of low and high risks are more convex than in the absence of ambiguity. This generates two sets of effects: a “coverage effect” and an “ambiguity effect.”¹⁴ First, the change in the shape of the high risk indifference curves

¹⁴ Crocker and Snow (2008) study the introduction of background risk when preferences are risk vulnerable. This also increases the aversion towards endogenous risks over wealth. They do not address ambiguity though.

relaxes the self-selection constraint. This increases the coverage in the separating low risk policy and decreases the critical value of λ . Second, the change in the shape of the low risk indifference curve makes it easier to attract the low risks away from the separating low risk policy. This increases the critical value of λ . The coverage effect makes low risks better off, the fact that they experience ambiguity makes them worse off. The change in welfare is determined by which of these effects is larger.

Let us formalize these ideas. We denote by C^{H0} and C^{L0} the RS policies for high and low risks, respectively, in the absence of ambiguity, i.e. when ambiguity is zero or, alternatively, if individuals are ambiguity neutral. We know that C^{H0} provides terminal wealth in both states of the world of $W^{H0} = W - \pi^H l$, and the low risk policy is determined by the binding self-selection constraint

$$u(W - \pi^H l) = U^H(W_N^{L0}, W_A^{L0}).$$

Notice that this condition is not affected if we apply Φ on both sides due to strict monotonicity. We want to determine the effect of ambiguity on the incentive compatibility constraint and as such on the level of coverage available to low risks. First, note that

$$\Phi(u(W - \pi^H l)) = \Phi(U^H(W_N^{L0}, W_A^{L0})) > E\{\Phi(\tilde{U}^H(W_N^{L0}, W_A^{L0}))\}, \quad (5.1)$$

due to ambiguity aversion. Second, note that the difference between the low risk and high risk expected utility, evaluated at C^{L0} is

$$U^L(W_N^{L0}, W_A^{L0}) - U^H(W_N^{L0}, W_A^{L0}) = (\pi^H - \pi^L)(u(W_N^{L0}) - u(W_A^{L0})) = \Delta\pi \cdot \Delta u^0.$$

With ambiguity aversion, the high risks still get the fairly priced full coverage, $C^{H*} = C^{H0}$. Consequently, the introduction of ambiguity has no impact on the welfare of high risks. We let $C^{L*} = (W_N^{L*}, W_A^{L*})$ denote the RS policy for the low risks in the presence of ambiguity. This policy satisfies the self-selection constraint

$$\Phi(u(W - \pi^H l)) = E\{\Phi(\tilde{U}^H(W_N^{L*}, W_A^{L*}))\}.$$

Define the ambiguity premium, φ^t , by $V^t(W_N, W_A) = \Phi(U^t(W_N, W_A) - \varphi^t)$. At the RS policies under ambiguity the difference in ex ante welfare of low and high risks measured in expected utility terms is given by

$$\begin{aligned} & \Phi^{-1}(V^L(W_N^{L*}, W_A^{L*})) - \Phi^{-1}(V^H(W_N^{L*}, W_A^{L*})) \\ &= U^L(W_N^{L*}, W_A^{L*}) - U^H(W_N^{L*}, W_A^{L*}) - (\varphi^L - \varphi^H) = \Delta\pi \cdot \Delta u^* - (\varphi^{L*} - \varphi^{H*}). \end{aligned}$$

From (5.1), the self-selection constraint is relaxed by the introduction of ambiguity, and as a result C^{L*} provides more coverage than C^{L0} . It follows that $\Delta u^* < \Delta u^0$, and therefore

$$\begin{aligned} & \Phi^{-1}(V^L(W_N^{L*}, W_A^{L*})) - \Phi^{-1}(V^H(W_N^{L*}, W_A^{L*})) \\ & < U^L(W_N^{L0}, W_A^{L0}) - U^H(W_N^{L0}, W_A^{L0}) - (\varphi^{L*} - \varphi^{H*}). \end{aligned}$$

From the two self-selection constraints (the one with and the one without ambiguity) we obtain that $\Phi^{-1}(V^H(W_N^{L*}, W_A^{L*})) = u(W - \pi^H l) = U^H(W_N^{L0}, W_A^{L0})$, so that

$$\Phi^{-1}(V^L(W_N^{L*}, W_A^{L*})) < U^L(W_N^{L0}, W_A^{L0}) - (\varphi^{L*} - \varphi^{H*}).$$

If ambiguity aversion is increasing or constant, then $(\varphi^{L*} - \varphi^{H*}) \geq 0$, and the low risks are worse off in the presence of ambiguity. If ambiguity aversion is decreasing, then $(\varphi^{L*} - \varphi^{H*}) < 0$, and the low risks may be better off in the presence of ambiguity than in its absence. This may occur if ambiguity aversions decreases “fast enough”. To analyze this possibility more rigorously, we consider the effect of the introduction of a small level of ambiguity. We let $\tilde{\pi} = \pi + r\tilde{e}$, and analyze the effect of a marginal increase r above 0 on low risk ex ante welfare. Define

$$H(r) = E\{\Phi((\pi + r\tilde{e})u(W_A) + (1 - \pi - r\tilde{e})u(W_N))\}$$

Then $H'(r) = E\{\Phi'(\tilde{U})\tilde{e}\Delta u\} = 0$ when evaluated at $r = 0$. If $H''(0)$ is positive, then $H'(r)$ is increasing at 0 and conversely if $H''(0)$ is negative. Straightforward but tedious manipulation shows that $H''(0) > 0$ if, and only if,

$$\psi A_\Phi(U^H) - A_\Phi(U^L) > 0 \quad (5.2)$$

where $A_\Phi(\cdot) = -\Phi''(\cdot)/\Phi'(\cdot)$ denotes the coefficient of absolute ambiguity aversion and

$$\psi = \frac{\pi^L(1 - \pi^L)(u'(W_A^{L0}) - u'(W_N^{L0}))}{(1 - \pi^L)\pi^H u'(W_A^{L0}) - (1 - \pi^H)\pi^L u'(W_N^{L0})}.$$

Observe that ψ depends only on loss probabilities and risk preferences and that $\psi < 1$. The inequality in (5.2) makes precise the degree to which the decrease in absolute ambiguity aversion is fast enough to make the low risks better off. Collecting these results leads to

Proposition 6: Assume λ is large enough that the RS equilibrium exists. (a) The introduction of ambiguity has no effect on the welfare of the high risks. (b) If preferences satisfy non-decreasing absolute ambiguity aversion, the introduction of ambiguity makes the low risks worse off. (c) If preferences satisfy decreasing absolute ambiguity aversion, the introduction of ambiguity makes low risks better off if (4.2) holds, otherwise they are worse off.

This result implies that the welfare effect of introducing ambiguity differs from the welfare effect of introducing background risk. Cocker and Snow (2008) show that the introduction of background risk always decreases welfare. Proposition 4 shows that there are some circumstances where the introduction of ambiguity improves welfare. What's more, policies that aim at reducing ambiguity are not necessarily welfare enhancing in the presence of asymmetric information.

6. Conclusions

Empirical research finds that there are both positive and negative correlations between coverage and ex-post risk in insurance markets, that is, there is both adverse and advantageous selection. Theoretical research attempting to explain these empirical results assumes a second dimension of unobserved heterogeneity, but assumes the underlying loss distributions are known. In this paper we assume risk is the single dimension of unobserved heterogeneity but there is ambiguity in the

sense that the loss probabilities are not known with certainty. We extend the Rothschild-Stiglitz (1976) model by assuming that there is ambiguity and that consumers have KMM smooth ambiguity-averse preferences. We make no other modifications to the Rothschild-Stiglitz model.

When consumers are ambiguity-averse, the indifference curves are still downward sloping and convex (in state space). Increases in ambiguity and increases in ambiguity aversion make consumers more averse to lotteries over wealth. Consumers still fully insure at actuarially fair prices. However ambiguity aversion raises the possibility that the single-crossing condition may not hold. We show that decreasing or constant absolute ambiguity aversion is sufficient for the single-crossing condition to hold. Increasing absolute ambiguity aversion is necessary but not sufficient for single-crossing to fail.

When single-crossing does not hold, equilibria with advantageous selection may potentially exist. However, we show that, under general assumptions, actuarially fair pricing implies that advantageous selection equilibria cannot occur. Also under general assumptions, we derive the conditions for second-best efficiency and provide a simple proof of uniqueness. The characterization is analogous to the expected utility case. The second-best contracts are characterized by adverse selection; advantageous selection is inefficient. We then discuss equilibria under the smooth model. If single crossing does not hold then there are Rothschild-Stiglitz equilibria with zero profits and with positive profits. The positive profit equilibrium is not robust to competition in menus of contracts. We then examine efficiency. We show that ambiguity shift the efficient contract curve upward; intuitively, low risks receive more coverage to compensate for the ambiguity. We then compare the Wilson-Myazaki-Spence (WMS) equilibria with and without ambiguity. Ambiguity may increase or decrease coverage for the low risks.

When single-crossing holds, ambiguity aversion has implications for the existence of the RS equilibrium. We show that ambiguity aversion can increase or decrease the critical proportion of high risks below which the Rothschild-Stiglitz equilibrium does not exist. Ambiguity aversion makes individuals act “as if” they are more risk averse, which shifts the equilibrium contract. The high risks’ increased risk aversion relaxes the self-selection constraint, so that low risks obtain more coverage. The low risks’ increased risk aversion makes it easier to attract them to a defecting contract. However, ex ante welfare of low risks might increase or decrease from the introduction of ambiguity. As such, although individuals’ behavior can be described as if risk aversion increases, results are not identical to those in Crocker and Snow (2008).

If there is an advantageous selection equilibrium due to ambiguous loss probabilities, then consumers must have sufficiently strong increasing ambiguity aversion and prices must be actuarially unfair. Non-increasing absolute ambiguity aversion and actuarially fair pricing are each sufficient conditions for adverse selection equilibria.

Appendix 1: Proof of Proposition 1

We first explore how $M(\pi)$ behaves locally. Define $\Delta u = u(W_N) - u(W_L)$, which is positive. $dM/d\pi$ is a fraction whose sign is determined by the sign of the numerator. After some simplifications this numerator can be written as

$$u'(W_N)u'(W_L)\{E\{\Phi'(\tilde{U})\}E\{\Phi'(\tilde{U})\} + \Delta u(E\{\Phi'(\tilde{U})\tilde{e}\}E\{\Phi''(\tilde{U})\} - E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{e}\})\}.$$

Note that $\tilde{U}$ and $-\Delta u\tilde{e}$ only differ by a positive constant; as such we can rewrite the expression in curly brackets as follows:

$$E\{\Phi'(\tilde{U})\}E\{\Phi'(\tilde{U})\} + E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{U}\} - E\{\Phi'(\tilde{U})\tilde{U}\}E\{\Phi''(\tilde{U})\}.$$

We allow $\tilde{e}$ to have a discrete, continuous or mixed distribution. After a change of variables, let $\tilde{U}$ be distributed in $[\underline{u}, \bar{u}]$ according to the cumulative distribution function $F(u)$. We can then expand the sum of the second and third term according to

$$\begin{aligned} \int_{\underline{u}}^{\bar{u}} \Phi'(u) f(u) du \int_{\underline{u}}^{\bar{u}} \Phi''(v) v dF(v) - \int_{\underline{u}}^{\bar{u}} \Phi'(u) u dF(u) \int_{\underline{u}}^{\bar{u}} \Phi''(v) dF(v) = \\ = \int_{\underline{u}}^{\bar{u}} \int_{\underline{u}}^{\bar{u}} (v - u) \Phi'(u) \Phi''(v) dF(u) dF(v). \end{aligned}$$

The integrals exist since the integrand is continuous. Rather than integrating over the entire square $[\underline{u}, \bar{u}] \times [\underline{u}, \bar{u}]$, we slice it up along the diagonal and integrate over one of the resulting triangles only. Due to the fact that

$$\int_{\underline{u}}^{\bar{u}} \int_{\{u > v\}} (v - u) \Phi'(u) \Phi''(v) dF(u) dF(v) = \int_{\underline{u}}^{\bar{u}} \int_{\{u \leq v\}} (u - v) \Phi'(v) \Phi''(u) dF(u) dF(v),$$

we obtain that

$$\begin{aligned}
& \int_{\underline{u}}^{\bar{u}} \int_{\underline{u}}^{\bar{u}} (v - u) \Phi'(u) \Phi''(v) dF(u) dF(v) = \\
& = \int_{\underline{u}}^{\bar{u}} \int_{\{u \leq v\}} (v - u) (\Phi'(u) \Phi''(v) - \Phi'(v) \Phi''(u)) dF(u) dF(v) \\
& = \int_{\underline{u}}^{\bar{u}} \int_{\{u \leq v\}} (v - u) \Phi'(v) \Phi'(u) (A_\Phi(u) - A_\Phi(v)) dF(u) dF(v),
\end{aligned}$$

where $A_\Phi(u) = -\Phi''(u)/\Phi'(u)$ is the index of absolute ambiguity aversion. Now on $\{u \leq v\}$ we have that $(v - u)$ is non-negative, and so is $(A_\Phi(u) - A_\Phi(v))$ as long as absolute ambiguity aversion is non-increasing. This shows that non-increasing absolute ambiguity aversion is sufficient to obtain that $dM/d\pi$ is positive. Given that π is fixed but arbitrary, this argument shows that non-increasing absolute ambiguity aversion implies that $dM/d\pi$ is positive for all $0 \leq \pi \leq 1$. As such it follows that for any choice of $0 < \pi^L < \pi^H < 1$, non-increasing absolute ambiguity aversion is sufficient for the single-crossing property to be satisfied whereas increasing absolute ambiguity aversion is necessary (but not sufficient) for the single-crossing property to fail.

Appendix 2: Single-Crossing with Multiplicative Ambiguity

In this Appendix, we derive the sufficient condition for single-crossing when ambiguity takes the multiplicative form, $\tilde{\pi} = (1 + \tilde{\epsilon})\pi$. As in the text, the objective function is

$$V(W_N, W_L) = E\{\Phi(\tilde{U})\} = E\{\Phi(\tilde{\pi}u(W_L) + (1 - \tilde{\pi})u(W_N))\}$$

and the marginal rate of substitution is given by

$$M(\pi) = \left. \frac{dW_L}{dW_N} \right|_{V=\text{const}} = - \frac{E\{\Phi'(\tilde{U})(1 - \tilde{\pi})\}u'(W_N)}{E\{\Phi'(\tilde{U})\tilde{\pi}\}u'(W_L)} < 0.$$

This can be rewritten as

$$M(\pi) = - \left(\frac{E\{\Phi'(\tilde{U})\}}{E\{\Phi'(\tilde{U})\tilde{\pi}\}} - 1 \right) \cdot \frac{u'(W_N)}{u'(W_L)}.$$

The numerator of the derivative of the bracketed expression with respect to π is, after some simplifications, given by:

$$\pi \Delta u \left(\begin{aligned} & E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{e}\} + E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{e}^2\} \\ & - E\{\Phi'(\tilde{U})\tilde{e}\}E\{\Phi''(\tilde{U})\} - E\{\Phi'(\tilde{U})\tilde{e}\}E\{\Phi''(\tilde{U})\tilde{e}\} \end{aligned} \right).$$

Using the fact that $\pi \tilde{e} \Delta u = U(\pi) - \tilde{U}$, the last expression can be rewritten as follows:

$$\begin{aligned} & E\{\Phi'(\tilde{U})\tilde{U}\}E\{\Phi''(\tilde{U})\} - E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{U}\} \\ & + \frac{U(\pi)}{\pi \Delta u} (E\{\Phi'(\tilde{U})\tilde{U}\}E\{\Phi''(\tilde{U})\} - E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{U}\}) \\ & + \frac{1}{\pi \Delta u} (E\{\Phi'(\tilde{U})\}E\{\Phi''(\tilde{U})\tilde{U}^2\} - E\{\Phi'(\tilde{U})\tilde{U}\}E\{\Phi''(\tilde{U})\tilde{U}\}). \end{aligned}$$

Under non-increasing absolute ambiguity aversion we know from the previous analysis that the expression in the first and the second line are negative. For the expression in the third line, we will employ the integral technique again:

$$\begin{aligned} & \int_{\underline{u}}^{\bar{u}} \Phi'(u) dF(u) \int_{\underline{u}}^{\bar{u}} \Phi''(v) v^2 dF(v) - \int_{\underline{u}}^{\bar{u}} \Phi'(u) u dF(u) \int_{\underline{u}}^{\bar{u}} \Phi''(v) v dF(v) = \\ & = \int_{\underline{u}}^{\bar{u}} \int_{\underline{u}}^{\bar{u}} v(v-u) \Phi'(u) \Phi''(v) dF(u) dF(v). \end{aligned}$$

Furthermore,

$$\int_{\underline{u}}^{\bar{u}} \int_{\{u>v\}} v(v-u) \Phi'(u) \Phi''(v) dF(u) dF(v) = \int_{\underline{u}}^{\bar{u}} \int_{\{u\leq v\}} u(u-v) \Phi'(v) \Phi''(u) dF(u) dF(v),$$

so that the previous integral becomes

$$\int_{\underline{u}}^{\bar{u}} \int_{\{u \leq v\}} (v - u) \Phi'(u) \Phi'(v) \Phi''(u) (uA_{\Phi}(u) - vA_{\Phi}(v)) dF(u) dF(v).$$

On $\{u \leq v\}$ we have that $(v - u)$ is non-negative and that $(uA_{\Phi}(u) - vA_{\Phi}(v))$ is non-positive as long as relative ambiguity aversion is non-decreasing. This shows that non-increasing absolute ambiguity aversion and non-decreasing relative ambiguity aversion together are sufficient to obtain that $dM/d\pi$ is positive for any $0 \leq \pi \leq 1$.¹⁵ Then, single-crossing will hold. Conversely, a necessary condition when single-crossing is not satisfied, is that either absolute ambiguity aversion is increasing or that relative ambiguity aversion is decreasing.

Appendix 3: Proof of Proposition 3

Since $\mathcal{V}^L$ and $\mathcal{V}^H$ are increasing, the resource constraint is binding, which proves (a). Proposition 2(a) shows that the high risks are fully insured, which proves (b).

The self-selection constraints cannot both be slack, for then the expected payoff of both types can be increased. We show that the high risk self-selection constraint (4.2) must be binding. Let (C^H, C^L) be a solution to the constrained maximization problem such that the resource constraint is binding. Suppose the high risk self-selection constraint is not binding, $\mathcal{V}^H(W^H, W^H) > \mathcal{V}^H(W_N^L, W_A^L)$, and $W_N^L > W_A^L$. Consider $C^{L'} = (W_N^L - \delta_1, W_A^L + \delta_2)$, where $\delta_2 = \frac{1-p^L}{p^L} \delta_1$. If δ_1 is small enough, then (4.2) is still slack. But $C^{L'}$ is a mean preserving contraction relative to C^L , so $\mathcal{V}^L(C^{L'}) > \mathcal{V}^L(C^L)$. Therefore, (C^H, C^L) cannot be a solution, which proves (c).

¹⁵ Notice that the two conditions are neither necessary nor sufficient for one another. One example that satisfies both of them is an exponential specification with $\Phi(x) = -\exp(-\alpha x)$, $\alpha > 0$.

Assume that $\mathcal{V}^H$ and $\mathcal{V}^L$ are differentiable. The Lagrangian for the maximization problem is

$$\begin{aligned}\mathcal{L} = & \mathcal{V}^L(W_N^L, W_A^L) + \mu^H[\mathcal{V}^H(W_N^H, W_A^H) - \mathcal{V}^H(W_N^L, W_A^L)] \\ & + \mu^L[\mathcal{V}^L(W_N^L, W_A^L) - \mathcal{V}^L(W_N^H, W_A^H)] \\ & \gamma[(W - \bar{\pi}l) - \lambda((1 - \pi^H)W_N^H + \pi^H W_A^H) - (1 - \lambda)((1 - \pi^L)W_N^L + \pi^L W_A^L)] \\ & \delta[\mathcal{V}^H(W_N^H, W_A^H) - \bar{\mathcal{V}}^H]\end{aligned}$$

where μ^H, μ^L, γ and δ are the Lagrangian multipliers. Denoting the partial derivatives by $\mathcal{V}_j^t$, the first order conditions are

$$\frac{\partial \mathcal{L}}{\partial W_N^L} = (1 + \mu^L)\mathcal{V}_N^L(W_N^L, W_A^L) - \mu^H\mathcal{V}_N^H(W_N^L, W_A^L) - \gamma(1 - \lambda)(1 - \pi^L) = 0 \quad (\text{A3.1})$$

$$\frac{\partial \mathcal{L}}{\partial W_A^L} = (1 + \mu^L)\mathcal{V}_A^L(W_N^L, W_A^L) - \mu^H\mathcal{V}_A^H(W_N^L, W_A^L) - \gamma(1 - \lambda)\pi^L = 0 \quad (\text{A3.2})$$

$$\frac{\partial \mathcal{L}}{\partial W_N^H} = (\delta + \mu^H)\mathcal{V}_N^H(W_N^H, W_A^H) - \mu^L\mathcal{V}_N^L(W_N^H, W_A^H) - \gamma\lambda(1 - \pi^H) = 0 \quad (\text{A3.3})$$

$$\frac{\partial \mathcal{L}}{\partial W_A^H} = (\delta + \mu^H)\mathcal{V}_A^H(W_N^H, W_A^H) - \mu^L\mathcal{V}_A^L(W_N^H, W_A^H) - \gamma\lambda\pi^H = 0 \quad (\text{A3.4})$$

along with the complementary slackness conditions.

If both self-selection constraints are binding, both types must be at the population pooled contract $\mathcal{C}^P$. The high risk utility constraint (4.5) must be set so that $\bar{\mathcal{V}}^H = \mathcal{V}^H(W^P, W^P)$. If $\bar{\mathcal{V}}^H < \mathcal{V}^H(W^P, W^P)$, the low risk self-selection constraint is slack and the high risk self-selection constraint is binding and we have $\mu^L = 0, \mu^H > 0$.

Equations (A3.1) and (A3.2) yield

$$\frac{\mathcal{V}_N^L(W_N^L, W_A^L)}{\mathcal{V}_A^L(W_N^L, W_A^L)} = \frac{\mu^H\mathcal{V}_N^H(W_N^L, W_A^L) + \gamma\lambda(1 - \pi^L)}{\mu^H\mathcal{V}_A^H(W_N^L, W_A^L) + \gamma\lambda\pi^L} \quad (\text{A3.5})$$

Using (A3.3) to eliminate the Lagrangian multiplier γ in (A3.5), we have

$$\frac{dW_A^L}{dW_N^L} = -\frac{\mathcal{V}_N^L(W_N^L, W_A^L)}{\mathcal{V}_L^L(W_N^L, W_A^L)} = -\frac{\mu^H \mathcal{V}_N^H(W_N^L, W_A^L) + [\lambda(1-\pi^H)]^{-1}(\mu^H + \delta)(1-\lambda)(1-\pi^L) \mathcal{V}_N^H(W^H, W^H)}{\mu^H \mathcal{V}_L^H(W_N^L, W_A^L) + [\lambda(1-\pi^H)]^{-1}(\mu^H + \delta)(1-\lambda)\pi^L \mathcal{V}_N^H(W^H, W^H)} \quad (\text{A3.6})$$

Rearranging yields (d) in the text.

Now suppose that there are two distinct solutions to the constrained maximization problem, $(\bar{C}^H, \bar{C}^L)$ and $(\hat{C}^H, \hat{C}^L)$. Then we have $\mathcal{V}^L(\bar{W}_N^L, \bar{W}_L^L) = \mathcal{V}^L(\hat{W}_N^L, \hat{W}_L^L)$. Since the FCL is downward sloping by (A3.6), one of the low risk contracts, say, $\hat{C}^L$, must have more coverage than the other low risk contract. But then the tax on the low risks must be higher, and, since net transfers must balance, the subsidy to the high risks must also be higher. This implies $\hat{W}^H > \bar{W}^H$, making the high risks are better off. The solution $(\hat{C}^H, \hat{C}^L)$ Pareto dominates $(\bar{C}^H, \bar{C}^L)$ and therefore is the unique solution, which proves (e).

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FIGURE 1
Rothschild-Stiglitz and Wilson-Miyazaki-Spence Equilibria

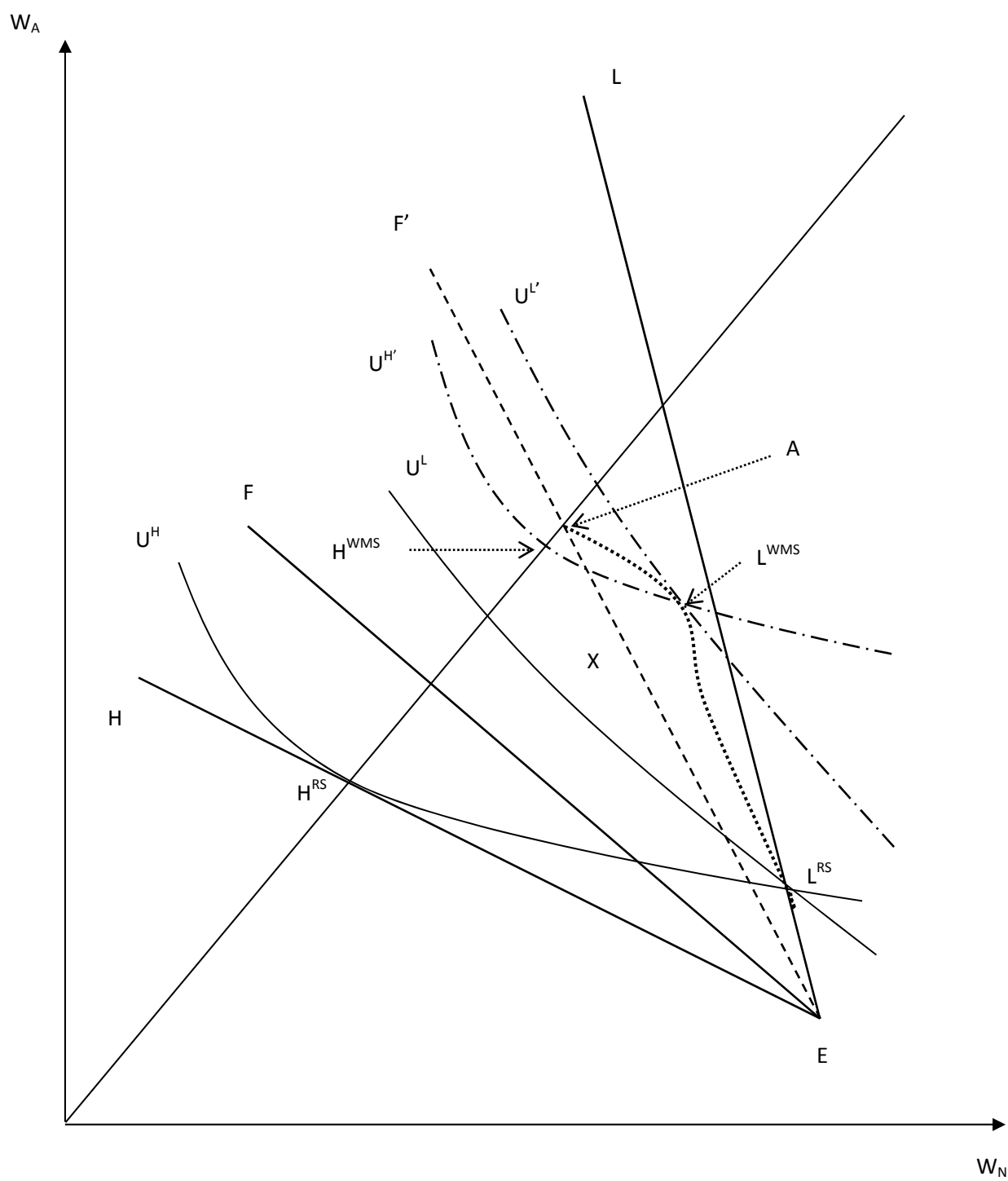


Figure 2
Positive Profit Rothschild-Stiglitz Equilibrium

