

A SIMPLE MODEL OF OPTIMAL CEO TURNOVER

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ABSTRACT

We examine the board of directors' problem of determining when to terminate a CEO. The optimal policy takes a simple form – the CEO is fired if profits fall below a critical value. The analysis yields the optimal probability of CEO turnover.

JEL Code: G34

Keywords: corporate governance, real options, optimal stopping

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1. Introduction

A substantial literature on the determinants of CEO turnover has developed over the last two decades. Early research includes Coughlin and Schmidt (1985), Warner, Watts and Wruck (1988) and Weisbach (1988); Brickley (2003) summarizes the main findings. While the subsequent literature contains extensive discussions of the determinants of turnover, the primary focus has been empirical. The literature has tended to rely on interpretations of principal-agent models (e.g., Holmstrom, 1979, Holmstrom and Milgrom, 1987, Stiglitz and Weiss, 1983) or, less frequently, matching models (e.g., Jovanovich, 1979, Rosen, 1982) as its theoretical basis.

In this paper, we develop a simple formal model of CEO turnover and tenure. That is, we examine the board of directors' problem of determining an policy for terminating the CEO that maximizes firm value. The basic premise of the analysis is that the decision to fire or retain a CEO can be viewed as an optimal stopping problem. That is, the decision to terminate a CEO is analogous to the real options problem of whether to invest in a project when there is an option to abandon or the decision to abandon a current project.¹ Most analyses of real options focus on the value of the project. However, the main focus of our analysis is on the timing of the decision.

The optimal policy for terminating the CEO takes a simple form – the CEO is fired if current profits fall below a critical value. The CEO's tenure is then the time until profits drop below the critical value, and the probability of CEO turnover at any point in time is the probability that profits have fallen below the cutoff. The analysis yields an

optimal probability of CEO turnover that, at least in principal, can be estimated empirically.

The next section develops the general model. Section 3 provides an example assuming profits follow Brownian motion. The last section provides brief concluding remarks.

2. The Model

The CEO generates a stream of profits $P(t)$, where $P(t)$ is some stochastic process. CEO compensation, C , will typically depend on the current level of profits and time. Compensation may also depend on the history of the process. For example, compensation might be tied to average profits over some period, or bonuses may depend on profits exceeding a certain level within a certain period. Since the board of directors determines compensation policy, CEO compensation will depend on the characteristics of the board. Then, in general, CEO compensation is $C(P, H_c, B, t)$, where H_c denotes the history of the process, and B denotes board characteristics.

The CEO can be terminated and replaced at cost S , where S is the cost of the severance package, the search for the new CEO, and other costs associated with replacing the CEO. The cost of replacing the CEO will depend on the current level of profits, the CEO's compensation and time. The cost of replacing the CEO will also depend on the firm's past performance, that is, on the history of the process. Since the decision to terminate the CEO is made by the board of directors, the cost of termination also depends on the characteristics of the board. For example, weak boards may find it more difficult (more costly) to terminate a CEO. Then in general, $S = S(P, C, H_s, B, t)$, where H_s

¹ See for example Dixit and Pindyck (1994, Ch. 7 and pp. 110-112) for analyses of the decision to invest with an abandonment option and the decision to abandon a project.

denotes the history.² If the CEO is terminated, the firm then hires a new CEO. Profits under the new CEO are generated by a new stochastic process, $\hat{P}$, with expected net present value $\hat{V}$. The payoff to the firm of terminating the CEO is then $\hat{V} - S$.

The problem for the board of directors is to determine a policy for terminating the CEO that maximizes the value of the firm.³ This is an optimal stopping problem. The board's optimal termination policy must satisfy the Bellman equation:

$$V = \max \{ \hat{V} - S, P - C + (1+r)^{-1} E_0[V] \}, \quad (1)$$

where V is the value of the firm net of CEO compensation, r is the discount rate and E_0 denotes the expectation given the information at $t = 0$. Under fairly general conditions, the optimal stopping/termination rule takes the form: "Terminate the CEO if $P(t) \leq P^*$ " where the reservation profit level P^* is determined as part of the solution to the problem.⁴ The reservation profit level is an absorbing boundary, since the CEO is terminated and the process for current profits is stopped if it ever reaches the boundary.

The tenure of the CEO, T , is then the time it takes until $P(t) \leq P^*$. This is the first passage time of the process $P(t)$ to the barrier P^* . For any individual CEO this is random. But, given the stochastic process for profits, the distribution of CEO tenure is the distribution of these first passage times, $F(t)$. The distribution $F(t)$ is interpreted as the probability that the CEO has been terminated by time t . Since $F(t)$ is derived from the value maximizing decision of the board, it gives the optimal probability of turnover. The

² The cost of replacing the CEO need not depend on the history of the process in the same way as compensation, so that $H_s \neq H_c$.

³ We assume the board of directors must make an immediate decision to hire/retain the CEO, and does not have the option to postpone the decision.

⁴ See, for example, Oksendal (1995, pp. 183-197).

stochastic process for profits and the rule for terminating the CEO together imply an optimal probability of CEO turnover that, at least in principle, we can estimate.

3. A Brownian Motion Example

We illustrate the problem with a simple example. We assume that the stochastic process for profits is (arithmetic) Brownian motion:

$$dP = \alpha dt + \sigma dz, \quad (2)$$

where z is a standard Weiner process. We assume that the CEO is compensated at the constant rate c . We also assume that both $\hat{V}$ and S are constant. Then the value of the firm is:

$$V_0 = (P_0 - c)/r + \alpha/r^2, \quad (3)$$

where we implicitly assume the CEO lives forever. The board of directors terminates the CEO if

$$P(t) \leq P^* = c + r[\hat{V} - S - \alpha/r^2]. \quad (4)$$

For our example, we can write $P(t) = P_0 + \alpha t + \sigma z(t)$, where $z(0) = 0$. Then $P(t) \leq P^*$ is equivalent to

$$z(t) \leq (P^* - P_0)/\sigma - (\alpha/\sigma)t. \quad (5)$$

The problem of determining the probability of CEO turnover is equivalent to the problem of finding the distribution of first passage times to a linear absorbing boundary for a standard Weiner process.⁵ We assume that $P_0 > P^*$ (otherwise the CEO is terminated immediately), and observe that this implies the intercept in (5) is negative. Then the distribution of first passage times is:

⁵ We can generalize the example slightly. Let $C = c_0 + c_1 t$, $S = s_0 + s_1 t$ and $\hat{V} = v_0 + v_1 t$. Then (5) still holds, with P^* replaced by $[c_0 + r(v_0 - s_0 - \alpha/r^2)]$ and α replaced by $\alpha - c_1 - r v_1 + r s_1$.

$$F(t) = 1 - \Phi\left(\frac{(P_0 - P^*) + \alpha t}{\sigma\sqrt{t}}\right) + \exp\left\{\frac{-2\alpha(P_0 - P^*)}{\sigma^2}\right\} \Phi\left(\frac{-(P_0 - P^*) + \alpha t}{\sigma\sqrt{t}}\right), t > 0, \quad (6)$$

where Φ is the standard normal distribution.⁶ This distribution is known as the Inverse Gaussian or Wald distribution.⁷ Given the assumptions of the example, (6) gives the optimal probability of CEO turnover.

If $\alpha \leq 0$, then the drift in profits is toward the reservation profit level P^* and profits must eventually fall below P^* . Formally, as $t \rightarrow \infty$, $F(t) \rightarrow 1$, so that the CEO is eventually terminated. For $\alpha < 0$, the expected tenure is $E\{T\} = (P^* - P_0)/\alpha$. However, if $\alpha > 0$, then the drift in profits is away from the reservation level and it is possible that profits never fall below P^* . In this case, as $t \rightarrow \infty$, $F(t) \rightarrow \exp\{-2\alpha(P_0 - P^*)/\sigma^2\} < 1$, and the distribution is said to be "defective." That is, if the drift in profits is positive (away from the barrier), then there is a strictly positive probability that the CEO is never terminated. Consequently, expected tenure is undefined for $\alpha > 0$. In either case, we have $\partial F(t)/\partial P^* > 0$; an increase in the reservation profit level increases the probability of termination at any given time.

From (6), an increase in CEO compensation, c , increases P^* , which leads to a higher probability of turnover. Similarly, an increase in the value of the firm under a new CEO, $\hat{V}$, or a decrease in the cost of searching for a new CEO, S , increases P^* . Finally, an increase in the discount rate, r , increases P^* .

The CEO can take actions, x , that affect the firm's flow of profits, so that the parameters of the stochastic process for profits depend on x . Good decisions are those

⁶ See Ingersoll (1987 pp. 350-354) or Salminen (1988).

⁷ If the stochastic process is geometric Brownian motion and the barrier is exponential, $b_0 \exp(b_1 t)$, then the first passage times also have an Inverse Gaussian distribution.

that increase the value of the firm. In our Brownian motion example, we can take the drift α as an increasing function of CEO actions, $\alpha(x)$, where $\alpha' > 0$. Then $\partial V_0 / \partial x = \alpha'(x)/r^2 > 0$ and $\partial P^* / \partial x = -\alpha'(x)/r^2 < 0$, so that actions that increase the value of the firm reduce the reservation level of profits. For $\alpha < 0$, actions that increase V_0 also increase expected tenure. However, we have $\partial F / \partial x = (\partial F / \partial P^*)(\partial P^* / \partial \alpha)\alpha' + (\partial F / \partial \alpha)\alpha' \gtrless 0$. The first term is negative, but the sign of $\partial F / \partial \alpha$ is ambiguous.

The CEO can also take actions, e , that entrench the CEO by increasing the cost of termination. This is Shliefer and Vishny's (1989) definition of entrenchment. Hermalin and Weisbach (1998) argue that, over time, the CEO's bargaining power with the board of directors increases, so the board becomes less likely to terminate the CEO. That is, the actions that the CEO takes to increase bargaining power also increase the cost of termination. Both of these views imply that we can take S to be an increasing function of e . In our Brownian motion example, we have $\partial P^* / \partial e = -rS'(e) < 0$. Then $\partial E\{T\} / \partial e = -rS'(e)/\alpha > 0$ for $\alpha < 0$ and $\partial F / \partial e = (\partial F / \partial P^*)(\partial P^* / \partial e) < 0$; actions that entrench the CEO increase expected tenure and decrease the probability of termination.

4. Conclusion.

We analyze the board of directors' problem of when to terminate a CEO. The board's problem is an optimal stopping problem, analogous to the decision to abandon an investment project. The optimal policy takes a simple form – fire the CEO if profits fall below a (endogenous) critical level. The probability that profits fall below the cutoff gives the optimal probability of turnover. We derive the probability of turnover when profits follow Brownian motion and provide some comparative statics results.

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