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Changes in Risk and Kinked Payoffs:
The Case of Initial Public Offerings With Bankruptcy Risk

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Abstract

This paper analyzes the effects of changes in risk when the individual's wealth has a kink and is piece-wise linear in the risk. The specific problem considered is that of a risk averse entrepreneur considering an IPO, explicitly taking account of the possibility that the firm may become bankrupt after the IPO. The possibility of bankruptcy affects how changes in risk affect the entrepreneur's welfare and the decision about the optimal share of the firm to retain. While standard arguments do not apply directly, there are two ways they can be adapted to the case where wealth has a kink.

JEL Classification: D81, G24, G33

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Introduction.

In this paper I examine the effects of changes in risk for a model where the individual's wealth has a kink and is piece-wise linear in the source of uncertainty. The kink in wealth is the result of the limited liability protection afforded by bankruptcy. An important consequence of limited liability is that standard results may not hold.¹ Changes in risk that would make the individual worse off or would decrease investment in the risky asset when wealth is linear in the risk may make the individual better off or increase investment when wealth is piece-wise linear. The objective of the paper is to determine the types of changes in risk that make the individual worse off and that decrease investment in the risky asset. I show that, while standard arguments developed for the linear model do not apply directly, they can be adapted to the case where wealth has a kink.

For concreteness and to motivate the analysis, I examine the problem of a risk averse entrepreneur considering an initial public offering (IPO). There is, of course, a substantial theoretical and empirical literature on IPOs.² However, the possibility that the firm may become bankrupt following the IPO is rarely discussed in the literature. Brown (1970), analyzing 257 IPOs between 1948 and 1955, reports that 17 percent fail within ten years. Platt (1995), analyzing a sample of IPOs from the 1980s, estimates that 6 percent become bankrupt within 3 years. Demers and Joos (2005) report a spike in

¹ See Gollier (2001) for a review of the literature.

² See Jenkinson and Ljungqvist (2001) and Ritter and Welch (2002) for reviews of the literature.

bankruptcies of newly public firms in 2001. Howton (2006) reports that, of 290 IPOs in 1997, 58 (20%) failed within five years.³ As recent experience, especially with dot.coms makes clear, firms can and do become bankrupt after their IPOs.⁴ The possibility that the firm may become bankrupt should be expected to affect the entrepreneur's decision making. Chalmers, Dann and Harford (2002) provide evidence that directors and officers consider the possibility that the firms may perform poorly after the IPO.

Several other problems in which wealth is piece-wise linear in the source of uncertainty have been discussed in the literature. Since wealth is piece-wise linear in the risk, the model here is a special case of the model in Kanbur (1982). Kanbur's main conclusion, that standard results do not hold when there is a kink in the payoff function, motivates the analysis in this paper. The entrepreneur's problem analyzed here is most closely related to problems where the kink in wealth arises from limited liability.⁵ In the "judgement proof" problem, the kink in wealth arises from the fact that individuals can declare bankruptcy and pay less than the full amount of liability judgements (Shavell, 1986). As a result, there is a critical value of wealth below which the individual will not buy liability insurance. Gollier, Koehl and Rochet (GKR, 1997), using a model similar to

³ Other researchers define failure as delisting for negative reasons. Hensler, Rutherford and Singer (1997) analyze 741 Nasdaq IPOs during 1975-1984. They find that 55 percent were delisted by 1992, and that delisted firms traded for an average of 5 years. Jain and Kini (1999, 2000) analyze 877 IPOs during 1977-1990, reporting that 14 percent were delisted within 5 years and another 17 percent merged. Peristiani and Hong (2004) document the trend in delistings during 1980-2001. They report that firms that had gone public within the preceeding five years accounted for over two-thirds of delistings. Fama and French (2004) report that 40 percent of 1980-1990 IPOs were delisted for cause within 10 years. Cockburn and Wagner (2010) analyze 356 IPOs of Internet firms between 1998 and 2001. They find that nearly two-thirds were delisted by early 2005. Chen, Rutherford and Wang (2013) analyze 5,367 IPOs from 1970 to 2011, finding that 25 percent delisted due to liquidation.

⁴ Two examples are eToys, which had its IPO in May 1999 and filed for bankruptcy in May 2001 and Global Crossing (IPO, August 1998, bankrupt January 2002). Not all failures are tech firms, for example, Boston Chicken (IPO November 1993, bankrupt October 1998). See Warren and Westbrook (1999) on the characteristics of firms in bankruptcy.

⁵ Two other problems where the wealth is piece-wise linear are the optimal deductible insurance policy (e.g., Schlesinger, 1981, Demers and Demers, 1991, Eeckhoudt, Gollier and Schlesinger, 1991 and Meyer

the one employed here, analyze an investment problem with bankruptcy. In GKR's model, bankruptcy occurs when the investor's wealth is zero, so it can be interpreted as a model of investment with personal bankruptcy. GKR show that limited liability increases the optimal investment in the risky asset. GKR also show that if wealth falls below a critical value, the investor will maximize their exposure to risk and "bet for resurrection." The model analyzed here is one of business bankruptcy – the firm in which the entrepreneur retains a share may become bankrupt but this need not leave the entrepreneur with zero wealth. In GKR's model the scale of investment affects the probability of bankruptcy, but not the individual's wealth if bankruptcy occurs. In the model examined here, the scale of investment affects the entrepreneur's wealth if bankruptcy occurs but not the probability of bankruptcy.

The kink in the entrepreneur's wealth leads to a number of questions about the effects of changes in risk on the entrepreneur's decisions. The first question is whether an increase in risk makes the individual better off or worse off. This can be viewed as a problem of the agency cost of debt - does the entrepreneur benefit from an increase in the riskiness of the firm's assets? For risk neutral investors in leveraged firms, an increase in the riskiness of assets makes them better off. In the standard portfolio model, an increase in risk always makes a risk-averse investor worse off. In kinked payoff model analyzed here, the entrepreneur may be either better off or worse off. Another question is how a change in risk affects the entrepreneur's decision about the share of the firm to retain. This problem has been extensively examined in the standard portfolio model.

and Meyer, 1999) and the "newsboy" problem (e.g., Eeckhoudt, Gollier and Schlesinger, 1995, Khouji, 1999, Ibarra-Salazar, 2005). In both of these problems the kink in wealth is endogenous.

Determining the effects of changes in risk typically requires restricting either the agent's preferences or the types of changes in risk. I show that restricting preferences is unlikely to yield the desired results. Two alternative approaches to answering these questions are examined in this paper. The first approach is to find different restrictions on the changes in risk that yield the desired results. Under this approach, I analyze the distribution of asset values conditional on solvency. The second approach is to transform the problem into one that is linear in the source of risk. Under this approach, I analyze the distribution of capital gains. Both approaches lead to sensible sufficient conditions, or necessary and sufficient conditions, for a change in risk to make the individual worse off or to decrease the individual's exposure to risk.

Section 2 describes the basic model. Sections 3 and 4 discuss the welfare effects and comparative statics effects of changes in risk. Section 5 provides the comparative statics results for the other parameters of the model. Section 6 provides brief concluding remarks.

2. The Basic Model.

The entrepreneur is an expected utility maximizer with a thrice continuously differentiable von-Neumann-Morgenstern utility function, $u(\cdot)$, over final wealth, W . Individuals' preferences are assumed to be nonsatiated and risk averse, i.e., $u' > 0$, $u'' < 0$, and satisfy the Inada conditions. The Arrow-Pratt measure of absolute risk aversion is $a = -u''/u'$, preferences are decreasing absolute risk averse (DARA) if $a' < 0$. The

entrepreneur's (non-stochastic) outside wealth, unrelated to the performance of the firm, is $w > 0$.⁶

The entrepreneur currently owns the firm entirely and is considering selling a fractional share $1 - \theta$ of the firm in an IPO and retaining the share θ , where $0 \leq \theta \leq 1$. The entrepreneur's motive for considering the IPO is to diversify her portfolio.⁷ The entrepreneur has a firm commitment offering from an investment bank such that the price per share is p . The firm has a fixed total of N shares, so full ownership of the firm's equity can be purchased for $P = pN$. The value of the firm's assets, denoted x , is risky. The value of assets has distribution F which has a continuous density f , and support in $[0, m]$. The expected value of assets is $\mu > 0$.

The face value of the firm's debt is d , and is exogenous. One can think of the entrepreneur as having previously borrowed to finance the purchase of the risky asset, with d being the amount to be repaid. It is assumed that d is in the interior of the support of F so the probability of bankruptcy is positive. Since the firm has the option to default, equity is a call option on the assets and the value of equity at the maturity of the debt is $\max(x - d, 0)$ (Black and Scholes, 1973). The proceeds of the sale are invested in the risk-free asset; to simplify the notation, the risk-free rate is assumed to be zero. Then the entrepreneur's final wealth is $W = w + (1 - \theta)P + \theta \max(x - d, 0)$ and

$$(2.1) \quad U(\theta) = E\{u(w + (1 - \theta)P + \theta \max(x - d, 0))\}$$

⁶ Assuming that outside wealth is subject to a mean zero additive background risk or mean one multiplicative background risk that is independent of the value of the firm's assets does not change any of the results in the paper. Letting z be the background risk and $u(w, z)$ be utility with the additive or multiplicative background risk, then the derived utility function $v(w) = E_z\{u(w, z)\}$ is increasing and concave if u is. See Kimball (1993) on the additive case and Franke, Schlesinger and Stapleton (2006) on the multiplicative case.

⁷ Other potential motives for considering an IPO, such as raising equity for investment opportunities or to pay off debt are not examined in this paper.

gives the entrepreneur's expected utility. This model reduces to the standard linear portfolio model if $d = 0$ or if $w > d$ and the entrepreneur has unlimited liability.

Alternatively, consider the problem of an investor who is not well diversified deciding whether to buy a share, θ , of a single firm. Since the investor is not well diversified, the possibility that the firm may become bankrupt is important. The equity of the firm can be purchased for P and the firm has debt d . Then the individual's final wealth is $W = w - \theta P + \theta \max(x - d, 0)$, and

$$(2.2) \quad U(\theta) = E \{u(w - \theta P + \theta \max(x - d, 0))\}$$

gives the investor's expected utility. Except for the investor's wealth, this is the same as the entrepreneur's problem. This describes, for example, the situation of many individuals investing in small businesses.

3. Welfare Effects of Changes in Risk

I want to analyze the effects of changes in the distribution of the value of the assets. In this section, I am concerned with the issue of whether an increase in risk makes the entrepreneur better off or worse off.

Take G to be the new distribution of x , letting U_F and U_G denote expected utility. Throughout the paper G will be "worse" than F . Suppose first that G is an increase in risk compared to F . Let $S(x) = \int_0^x [G(z) - F(z)]dz$. Then G is an increase in risk compared to F if $S(0) = S(m) = 0$ and $S(x) \geq 0$ on $[0, m]$. As is well known, the individual has lower expected utility under G if, and only if, u is concave in x (Rothschild and Stiglitz, 1970).

The following result implies that, if the firm can become bankrupt, the change from F to the riskier distribution G need not make the entrepreneur worse off.

Proposition 1: Assume the owner of share $\theta > 0$ of the firm is risk averse and not well diversified. If the firm has debt $d > 0$ then, in general, u is neither concave nor convex in x .

Proof: An example is sufficient to prove the claim. Regarding u as a function of the value of assets, and assuming that utility has constant absolute risk aversion, $h(x) = (-1/\alpha)\exp[-\alpha(w + (1 - \theta)P + \theta\max(x - d, 0))]$. This can be written as $h(x) = K\exp[-\alpha\theta\max(x - d, 0)]$, where $K = (-1/\alpha)\exp[-\alpha(w + (1 - \theta)P)] < 0$. Let $x^* = (1 - t)x' + tx''$ for $0 \leq t \leq 1$. We need to show that

$$(3.1) \quad h(x^*) - [(1 - t)h(x') + th(x'')] \geq 0$$

holds for some value of x', x'' and t , and that the inequality is reversed for other values.

First, assume that $x'' > x' > d$. Then $h(x') = K\exp[-\alpha\theta(x' - d)]$, $h(x'') = K\exp[-\alpha\theta(x'' - d)]$ and $h(x^*) = K\exp[-\alpha\theta(x^* - d)]$. Upon substituting and rearranging, (3.1) holds since $\exp[-\alpha x^*] - \{(1 - t)\exp[-\alpha\theta x'] + t\exp[-\alpha\theta x'']\} < 0$. This shows that h is not convex in x .

Now assume that $x' < d < x''$. Then $x^* \leq d$ if $t \leq t^* = (d - x')/(x'' - x')$. For $t \leq t^*$, we have $h(x') = h(x^*) = K$ and $h(x'') = K\exp[-\alpha\theta(x'' - d)]$. Substituting and rearranging, (3.1) holds if, and only if, $1 - \exp[-\alpha\theta(x'' - d)] \leq 0$. But $x'' > d$, so that this expression is strictly positive. This implies that h is not concave in x . Finally, for $x' < x'' < d$, (3.1) holds as an equality. ||

Proposition 1 rests on the fact that the entrepreneur's wealth, and therefore utility, are constant in all of the states of the world where the firm is bankrupt. The kink in the

entrepreneur's wealth resulting from the option to default creates a kink in the entrepreneur's utility as a function of the value of the firm's assets.

An important point to note is that the argument does not depend on the degree of risk aversion. This implies that the problem cannot be avoided by assuming that economic agents are "sufficiently" risk averse.⁸ Most commonly used utility functions are convex combinations of negative exponential utility functions (Brockett and Golden, 1987, Thistle, 1993). This suggests it will be difficult to find reasonable restrictions on preferences for which Proposition 1 does not hold.

Proposition 1 implies that the entrepreneur may prefer either the riskier or the less risky alternative. This leads to the question of the conditions under which the entrepreneur chooses the less risky alternative. There are two approaches to answering this question.

The first approach is to impose restrictions on F and G . Let $F^+(x)$ denote the conditional distribution $F(x | x \geq d)$ and similarly for G^+ . Letting $S^+(x) = \int_d^x [G^+(z) - F^+(z)]dz$, then G is conditionally riskier than F if $S^+(d) = S^+(m) = 0$ and $S^+(x) \geq 0$ on $[d, m]$.

Proposition 2: Assume the firm has debt $d > 0$. If
 (3.2) (a) $G(d) \geq F(d)$ and
 (b) $S^+(d) = S^+(m) = 0$ and $S^+(x) \geq 0$ on $[d, m]$,
 then $U_F(\theta) \geq U_G(\theta)$.

Proof: Rewrite $U_F(\theta)$ as

$$(3.3) \quad U_F(\theta) = u(w + (1 - \theta)P)F(d) \\ + [1 - F(d)] \int_d^m u(w + (1 - \theta)P + \theta(x - d))f^+(x)dx$$

Eq. (3.2)(a) states that bankruptcy is no more likely under F . Eq. (3.2)(b) implies that, conditional on $x \geq d$, expected utility is at least as high under F as G . Combining these results, $U_F(\theta) \geq U_G(\theta)$. $\parallel$

The assumption that G is riskier than F in the sense of Rothschild and Stiglitz is neither necessary nor sufficient for G to be conditionally riskier than F . Also, the condition $S^+(m) = 0$ implies that the expected values of assets, conditional on solvency, are equal. It does not imply that the unconditional expected values of assets are equal. It is possible for the unconditional expected value to be higher under G than F .

Proposition 2 yields some insight into the conditions under which an increase in risk makes the individual better off. First, the probability of bankruptcy could be lower under G . Condition (3.2)(b) is equivalent to requiring $E\{u|x \geq d\}$ to be higher under F than G for all increasing concave u , so the other possibility is that G raises expected utility in those states of the world where the firm is not bankrupt. If the shift to the riskier distribution G increases expected utility for all increasing concave u , then one of the inequalities in (3.2) must be reversed. Put differently, the conditions in (3.2) are necessary in the sense that if $U_F > U_G$ for all increasing concave u , then at least one of the inequalities (3.2)(a) or (3.2)(b) must hold.

The second approach to determining when the entrepreneur will choose the less risky distribution is to transform the problem into a linear problem and then derive the restrictions on F and G from the inverse transformation. Let

$$(3.4) \quad y = \varphi(x) = \max(x - d, 0) - P;$$

⁸ Let $h(x) = u(w + v(x))$, where v is increasing, convex and twice continuously differentiable. Then $h'' = -$

y is the ex post capital gain from buying at the IPO price. Then expected utility can be written as

$$(3.5) \quad U(\theta) = E\{u(w + P + \theta y)\}.$$

The distribution of y is $F_y(-P) = F_x(d) > 0$ and $F_y(y) = F_x(y + d + P)$ for $y \in (-P, m - d - P]$. That is, the problem is transformed from one in which wealth is piece-wise linear and convex into a problem in which wealth is linear in the risk, but the distribution has a mass point at $y = -P$. The transformation immediately leads to the following result:

Proposition 3: Assume the firm has debt $d > 0$. Let

$$(3.6) \quad S_y(-P) = S_y(m - d - P) = 0.$$

Then

$$(3.7) \quad S_y(y) \geq 0 \text{ on } [-P, m - d - P]$$

if, and only if, $U_F(\theta) \geq U_G(\theta)$ for all increasing concave u .

Propositions 2 and 3 both answer the question of when the entrepreneur will choose the less risky distribution. The answers given by Propositions 2 and 3 are almost the same. The difference is that Proposition 2 gives a sufficient condition while Proposition 3 gives a necessary and sufficient condition. The difference is due to the fact that $U_F(\theta) \geq U_G(\theta)$ and either (3.2)(b) or (3.7) do not imply that the bankruptcy probability is higher under G . However, if $G(d) \geq F(d)$, then G is conditionally riskier than F if, and only if, G_y is riskier than F_y . The condition $S_y(-P) = 0$ is equivalent to requiring equality of the bankruptcy probabilities, $G(d) = F(d)$. Also, observe that both of the conditions $S^+(m) = 0$ and $S_y(m - d - P) = 0$ imply that expected capital gains, and therefore that the value of the firm's equity, are equal under both distributions.

$[a_u + a_v]u'v' < 0$ if $a_u > |a_v|$, that is, if u is sufficiently risk averse.

4. Comparative Static Effects of Changes in Risk

In this section, I am concerned with the entrepreneur's optimal retention and with how a change in the distribution affects the optimal decision.

A. Optimal retention. The entrepreneur's problem is to determine the share of the firm to retain. The first order condition is

$$(4.1) \quad U'(\theta) = -u'(w + (1 - \theta)P)PF(d) + \int_d^m u'(w + (1 - \theta)P + \theta(x - d))[x - d - P]f(x)dx$$

The two terms are the effects of increasing retention when the firm becomes bankrupt and when it remains solvent. The first order condition can also be written in terms of capital gains as

$$(4.2) \quad U'(\theta) = E\{u'(w + P + \theta y)y\} = 0.$$

Since U is concave in θ , the first order condition is necessary and sufficient for a maximum. We let θ^* denote the solution to the maximization problem.

In the standard portfolio problem, a necessary and sufficient condition for investment in the risky asset is that the expected return on the risky asset exceeds the risk-free rate. In the model here, this condition is $E\{y\} > 0$. Under the assumptions of this model, the risk-neutral value of equity is $V = E\{\max(x - d, 0)\}$. Then $E\{y\} = V - P$ and the necessary and sufficient condition for an interior solution is $V > P$. This immediately leads to the following result:

Proposition 4: $V > P$ if, and only if, $\theta^* > 0$.

That is, the entrepreneur will retain a share of the firm if, and only if, the IPO is underpriced.⁹

Another question is how the option to default affects the entrepreneur's optimal retention. With unlimited liability, the entrepreneur's expected utility is

$$(4.3) \quad \bar{U}(\theta) = E\{u(w + (1 - \theta)P + \theta(x - d))\}$$

and the first order condition can be written as

$$(4.4) \quad \bar{U}'(\theta) = \int_0^d u'(w + (1 - \theta)P + \theta(x - d))(x - d - P)f(x)dx \\ + \int_d^m u'(w + (1 - \theta)P + \theta(x - d))(x - d - P)f(x)dx.$$

Let $\bar{\theta}$ denote the solution to the maximization problem under unlimited liability. This leads to the following result:

Proposition 5: If $\theta^* > 0$, then $\bar{\theta} < \theta^*$.

Proof: Comparing the expressions in (4.1) and (4.4), $\bar{U}'(\theta) < U'(\theta)$ for all $\theta > 0$. ||

That is, the entrepreneur's retention is greater under limited liability. Similar results are obtained by Shavell (1986) and Gollier, Kohl and Rochet (1997).

B. Changes in distributions. Gollier (1995) gives the general necessary and sufficient condition for a change in the distribution to decrease the level of investment.¹⁰ Gollier's result for the linear model is sufficient for the analysis here. Let $W = w + \alpha z$, suppose the distribution shifts from F_1 to F_2 , and let α_1 (α_2) maximize expected utility given u and F_1

⁹ Note that this result does not explain IPO underpricing, which is an equilibrium phenomenon. The result is consistent with the facts that on average, IPOs are underpriced and that insiders retain substantial equity positions in the firm after the IPO.

¹⁰ In the general case, Gollier assumes that final wealth $W(x, \theta)$ is twice differentiable in (x, θ) and concave in θ .

(F_2). Define $T_1(z) = \int_0^y t dF_1(t)$ and $T_2(z) = \int_0^y t dF_2(t)$. Then the distribution F_2 is said to be "centrally riskier" than F_1 if there is a real number γ such that $\gamma T_1(z) - T_2(z) \geq 0$ for all z . Gollier proves the following important result for models where wealth is linear in the risk:

Theorem: (Gollier, 1995): $\exists \gamma$ such that $\gamma T_1(z) - T_2(z) \geq 0$, for all z , if, and only if, $\alpha_2 \leq \alpha_1$ for all increasing concave u .

Further, if, as here, u is assumed to be smooth, if $\gamma T_1(z) - T_2(z) > 0$ for some z , and $\alpha_1 > 0$, then $\alpha_2 < \alpha_1$.

As Gollier points out, one distribution being centrally riskier than another is neither necessary nor sufficient for that distribution to be riskier in the sense of Rothschild and Stiglitz.¹¹ Greater central riskiness is an alternative definition of an increase in risk, namely, one that leads all risk averse individuals to decrease their exposure to risk. Which definition is more useful depends on the problem at hand. For the welfare analysis of the previous section, the Rothschild-Stiglitz definition is more useful. For the comparative statics analysis of this section, Gollier's definition is more useful.

Using the transformation to capital gains in (3.4), the entrepreneur's wealth is linear in y and Gollier's theorem can be applied. Let $T_{F_y}(y) = \int_0^y t dF_y(t)$ and define $T_{G_y}(y)$ similarly.

Proposition 6: Assume the firm has debt $d > 0$. $\exists \gamma$ such that $\gamma T_{F_y}(y) - T_{G_y}(y) \geq 0$, $\forall y \in [-P, m - d - P]$, if, and only if, $\theta_G^* \leq \theta_F^*$ for all increasing concave u .

If G_y is centrally riskier than F_y , the entrepreneur decreases the share of the firm that she retains.

This leaves the issue of the restrictions on F_x and G_x that lead to a decrease in the entrepreneur's retention. To derive the conditions on F_x and G_x , recall that $F_y(-P) = F_x(d) > 0$ and $F_y(x - d - P) = F_x(x)$ for $x \in (d, m]$.

Proposition 7: Assume the firm has debt $d > 0$. $\exists \gamma$ such that
 (4.5) (a) $\gamma F_x(d) \leq G_x(d)$ and
 (b) $\gamma T_{F_x}(x) - T_{G_x}(x) \geq 0, \forall x \in (d, m]$,
 if, and only if, $\theta_G^* \leq \theta_F^*$ for all increasing concave u .

Proof: First, observe that $\gamma T_{F_y}(-P) - T_{G_y}(-P) = \gamma T_{F_x}(d) - T_{G_x}(d) = -P[\gamma F_x(d) - G_x(d)]$ which is non-negative if and only if (4.5)(a) holds. Using the fact that $F_y(x - d - P) = F_x(x)$ for $x \in (d, m]$, the condition that $\gamma T_{F_x}(x) - T_{G_x}(x) \geq 0$ on $(d, m]$ is equivalent to $\gamma T_{F_y}(x - d - P) - T_{G_y}(x - d - P) \geq 0, \forall x \in (d, m]$. This in turn is equivalent to $\gamma T_{F_y}(y) - T_{G_y}(y) \geq 0, \forall y \in (-P, m - d - P]$. Taken together, (4.5)(a) and (b) are equivalent to G_y being centrally riskier than F_y . The conclusion then follows from Gollier's theorem. ||

Observe that it is not true that G_x centrally riskier than F_x implies that the optimal retention by the entrepreneur decreases. Since the asset values in $[0, d)$ don't affect the decision, G_x needs to be centrally riskier than F_x on $(d, m]$, the range of asset values for which the firm is solvent. However, the value of γ is constrained by the probability of bankruptcy and must satisfy $\gamma \leq G_x(d)/F_x(d)$.

¹¹ See Gollier (1995, 2001, pp. 92-93) on the relationship between "riskier" and "centrally riskier."

C. Single Crossing. In signaling models, an important consideration is whether the single crossing property holds. In the model here, the quality of the IPO is determined by the distribution of the value of the firm's assets, with higher quality assets having better distributions. To characterize quality, let λ be the quality parameter and write the density function for asset values as $f_x(x, \lambda)$. Assume that for $\lambda_H > \lambda_L$, $f_x(x, \lambda_H)/f_x(x, \lambda_L)$ is nondecreasing in x ; that is, the monotone likelihood ratio (MLR) property holds. Also, expanding the notation slightly, let $U(\theta, P, \lambda)$ denote the entrepreneur's expected utility (2.1) when the density of asset values is $f_x(x, \lambda)$.

Athey (2002) gives a sufficient condition for the single crossing property to hold in models that have the same structure as here. Let $H(a, b, \lambda) = \int h(a, b, z)f(z, \lambda)dz$. The single crossing property holds if $H_a/|H_b|$ is nondecreasing in λ (where subscripts denote derivatives and $H_b \neq 0$). Athey proves the following result:

Theorem 2: (Athey, 2002) Let (a) $h(a, b, z)$ satisfy the single crossing property in z and (b) $f(z, \lambda)$ satisfy the MLR property. Then (c) H satisfies the single crossing property in λ .

In fact, Athey proves a stronger result, namely, if (a) is assumed to hold, then conditions (b) and (c) are equivalent and if (b) holds, then conditions (a) and (c) are equivalent.

Given this result, all that is needed is to check whether u satisfies the single crossing property. From (2.1), $(\partial u / \partial \theta) / (\partial u / \partial P) = y / (1 - \theta)$, which is increasing in y . This leads to:

Proposition 8: If $f_y(y, \lambda)$ satisfies the MLR property, then $U(\theta, P, \lambda)$ satisfies the single crossing property.

Alternatively, since $y = \max(x - d, 0) - P$, it follows that u satisfies the single crossing condition in x . If $f_x(x, \lambda)$ satisfies the MLR property, then $f_y(y, \lambda)$ does also. It follows that if $f_x(x, \lambda)$ satisfies the MLR property, then $U(\theta, P, \lambda)$ satisfies the single crossing property. This also yields a comparative statics result. An increase in λ is a special case of a decrease in central riskiness, that is, $\lambda_H > \lambda_L$ implies $F(x, \lambda_L)$ is centrally riskier than $F(x, \lambda_H)$. Then $\partial \theta^* / \partial \lambda > 0$, the entrepreneur's retention is an increasing function of the quality of the assets.¹²

While a formal analysis is beyond the scope of this paper, Proposition 8 is suggestive. Suppose the entrepreneur's expected utility is given by (2.1) and that $f_x(x, \lambda)$ satisfies the MLR property. Suppose further that investment bankers are willing to undertake firm commitment offerings at prices that increase with the entrepreneur's retention, that is, according to an increasing function $P^*(\theta)$. Then it should be possible to prove that an equilibrium exists. It should also be possible to show that, in equilibrium, both the IPO price and the entrepreneur's retention are increasing in quality λ . The value of equity, V , is also increasing in λ . Proposition 4 implies that the IPO must be underpriced. An interesting question is the relationship between the degree of underpricing, $V - P$, and the quality of the IPO, λ , in equilibrium.

5. Comparative Statics Effects of Changes in Parameters

In this section, I am concerned with the effect that changes in the parameters (wealth, the firm's debt and the IPO price) have on the entrepreneur's optimal decision.

¹² See Gollier (2001, p. 93) and Athey (2002, Theorem 4).

The effect of a change in a parameter α on the entrepreneur's optimal retention is given by $\partial\theta^*/\partial\alpha = -(\partial U'(\theta^*)/\partial\alpha)/U''(\theta^*)$. Since U is concave in θ , $-U''$ is positive and the sign of $\partial\theta^*/\partial\alpha$ is determined by the sign of the numerator, $\partial U'(\theta^*)/\partial\alpha$. Also, observe that final wealth is non-decreasing in the value of assets. This implies that, under the assumption of decreasing absolute risk aversion, absolute risk aversion is non-increasing in x . In this section, preferences are assumed to be DARA.

First, consider the effect of an increase in outside wealth, w . Differentiating the first order condition in (4.1) or (4.2) yields

$$(5.1) \quad \partial U'(\theta^*)/\partial w = E\{u''(w + P + \theta y)y\}$$

Multiplying and dividing by $u'(w + P + \theta^* y)$, this becomes $E\{-au'y\} \geq 0$ and it follows that $\partial\theta^*/\partial w \geq 0$. This is the well-known result that, under DARA, an increase in wealth increases the optimal exposure to risk.

Now consider the effect of an increase in the firm's debt level, d . Differentiating the first order condition yields

$$(5.2) \quad \begin{aligned} \partial U'(\theta^*)/\partial d = & -\int_d^m u''(w + (1 - \theta)P + \theta(x - d))[x - d - P]f(x)dx \\ & - \int_d^m u'(w + (1 - \theta)P + \theta(x - d))f(x)dx \end{aligned}$$

The second term in (5.2) is negative, but the sign of the first term needs to be determined.

Adding and subtracting $u''(w + (1 - \theta^*)P)PF(d)$, the first term can be rewritten as

$$(5.3) \quad E\{-u''(w + P + \theta y)y\} + u''(w + (1 - \theta)P)PF(d).$$

The first term in this expression is the negative of the numerator of $\partial\theta^*/\partial w$ and is negative. The second term is also negative. This implies that both of the terms in (5.2)

are negative, and therefore $\partial\theta^*/\partial d \leq 0$. The more debt the firm has, the smaller the share of the firm that the entrepreneur is willing to retain.

Consider the effect of an increase in the IPO price, P . Differentiating the first order condition yields

$$(5.4) \quad \begin{aligned} \partial U'(\theta^*)/\partial P = & (1-\theta) \int_0^m u''(w + (1-\theta)P + \theta \max(x-d, 0)) [x-d-P] f(x) dx \\ & - \int_0^m u'(w + (1-\theta)P + \theta \max(x-d, 0)) f(x) dx \end{aligned}$$

The first term is $(1-\theta)$ times the numerator of $\partial\theta^*/\partial w$, and is positive, while the second term is negative. The effect of an increase in the price of the firm's equity is ambiguous. To see why, regard the entrepreneur as selling the entire firm for P , then buying a back a share. The higher selling price increases the entrepreneur's wealth, which tends to increase θ^* . The higher price also means that the entrepreneur pays more for the share of the firm bought back, which tends to decrease θ^* . The net effect is ambiguous.

For the small business investor with expected utility given by (2.2), the effect of a change in the price is unambiguous. Differentiating the first order condition yields

$$(5.5) \quad \begin{aligned} \partial U'(\theta^*)/\partial P = & -\theta \int_0^m u''(w + (1-\theta)P + \theta \max(x-d, 0)) [x-d-P] f(x) dx \\ & - \int_0^m u'(w + (1-\theta)P + \theta \max(x-d, 0)) f(x) dx \end{aligned}$$

The first term is $-\theta$ times the numerator of $\partial\theta^*/\partial w$ and is negative, while the second term is again negative, so that $\partial\theta^*/\partial P \leq 0$. For the investor, an increase in the price decreases the optimal share of the firm, that is, the investor's demand curve for ownership of the firm is downward sloping.

6. Conclusion

I analyze the effects of changes in risk when the individual's wealth is piece-wise linear in the source of uncertainty. The specific problem analyzed is that of a risk-averse entrepreneur contemplating an initial public offering, where the entrepreneur explicitly takes account of the possibility that the firm may become bankrupt after the IPO. The value of the firm's assets is risky and the kink in the entrepreneur's wealth is the result of the option to default on the firm's debt. While the piece-wise linearity of wealth implies that standard results do not hold in general, I show that standard arguments can be adapted to the case where there is a kink in wealth. Standard arguments can be adapted by analyzing the distribution of asset values, conditional on the solvency of the firm. Alternatively, standard arguments can be adapted by analyzing the distribution of capital gains.

I show that an increase in the riskiness of the firm's assets may make the entrepreneur either better off or worse off, and that this does not depend on the entrepreneur's degree of risk aversion. A sufficient condition for the entrepreneur to be worse off is for the probability of bankruptcy to increase and the distribution of asset values, conditional on remaining solvent, to become riskier. Alternatively, the entrepreneur is worse off if, and only if, the distribution of capital gains becomes riskier.

I also examine the conditions under which a change in risk decreases the share of the firm that the entrepreneur retains after the IPO. The share retained falls if, and only if, the distribution of capital gains becomes centrally riskier in the sense of Gollier (1995). An increase in the central riskiness of capital gains is equivalent to an increase in the central riskiness of asset values for the values where the firm is not bankrupt plus a

condition of the relative probabilities of bankruptcy. I show that, if the distribution of asset values satisfies the monotone likelihood ratio property, then expected utility satisfies the single-crossing property.

An interesting result is that a necessary and sufficient condition for the entrepreneur to retain a share of the firm after the IPO is that the IPO must be underpriced. This is the analog of the result in the standard portfolio model that risk-averse individuals invest in the risky asset if and only if there is a positive excess return. For the entrepreneur, the positive excess return comes in the form of an expected capital gain above the IPO price.

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