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Large Losses and Equilibrium in Insurance Markets

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ABSTRACT

We show that, if losses are larger than wealth, individuals will not insure if the loss probability is above a threshold. In an insurance market with adverse selection, if the high risks' loss probability is above the threshold, then no trade occurs at the Rothschild-Stiglitz equilibrium. Active trade in insurance requires cross-subsidization.

Keywords: adverse selection, contracts, no trade
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1. Introduction.

Individuals can face large potential losses that exceed their wealth. The median net worth of US households was $68,828 in 2011 (U.S. Census Bureau, 2014). It is easy see that a large medical expense or a large liability judgement could easily wipe out a household's net worth.[1] We show that, when losses exceed wealth, risk averse individuals will not insure if the probability of loss exceeds a threshold. We examine the implications of this for insurance markets with adverse selection. We show that if the high risks do not buy insurance then there is no trade in the insurance market. The Rothschild-Stiglitz equilibrium contracts for both high and low risks are the null contracts. This is the reverse of the "adverse selection death spiral" since it is the high risks rather than the low risks that drop out of the market.

Shavell (1986) and Sinn (1982) consider the case where losses exceed wealth. Shavell shows that, for a fixed loss probability, there is a wealth threshold where individuals do not insure if their wealth is below the threshold and fully insure if it is above the threshold. Sinn notes that whether the premium exceeds the maximum willingness to pay depends on the probability of loss but does not consider threshold values. Neither author considers the case of asymmetric information. Our result is the obverse of Shavell's. We show that, for fixed wealth,

[1] According to the American Automobile Association (2011), the average cost of an auto crash fatality was $6 million and the average cost of an injury was $126 thousand in 2009.

there is a probability threshold where individuals fully insure if the loss probability is below the threshold and do not insure if the loss probability is above the threshold.[2]

Hendren (2013, 2014) gives conditions under which the unique Rothschild-Stiglitz equilibrium contract in an insurance market with adverse selection is the null contract. The "no trade condition" is that lower risks are never willing to pay to be in a pool with higher risks. This leads to an adverse selection death spiral. If the no trade condition does not hold, then either there is an equilibrium where insurance is bought or there is no equilibrium. As Hendren (2014) points out the "equilibrium of market unraveling" (no trade) and the "unraveling of market equilibrium" (no equilibrium) are mutually exclusive.

We analyze the standard Rothschild-Stigliz (1976) model of adverse selection. The no trade condition does not hold in this model, so we would expect active trade in equilibrium. We relax Hendren's implicit assumption that losses do not exceed wealth. We show that if the high risks' loss probability exceeds the threshold, then the unique Rothschild-Stiglitz equilibrium contracts are the null contracts – there is no trade – or else there is no equilibrium. The no trade equilibrium is the result of the high risks dropping out of the market. The high risks do not buy insurance at an actuarially fair price because it covers losses they would not be able to pay if uninsured and is therefore too expensive. The low risks would be willing to buy insurance at their actuarially fair price if it provides a minimum level of coverage. The need to screen the high risks limits the coverage for the low risks to a level that is below this minimum and the low risks choose not to insure. We extend the analysis to the Wilson (1977)-Miyazaki (1977)-Spence (1978) equilibrium. If the proportion of high risks is large enough, then the equilibrium contracts

[2] Peter (2016) assumes losses are less than wealth and analyzes the threshold premium loading factor above which individuals do not insure.

are the null contracts and there is no trade. If the proportion of high risks is small then there is active trade in insurance. Active trade requires that the low risks subsidize the high risks.[3]

A key factor in our analysis is the assumption that final wealth cannot be negative so that a portion of any loss that is greater than an individual's wealth will be externalized. For example, the injured third party may not receive full compensation if the loss exceeds the wealth of the party liable for damages. Similarly, the medical expenses of uninsured or underinsured patients with insufficient wealth may ultimately be borne by medical providers. Our results may provide an explanation for why health insurance markets are not well-functioning for low income populations.[4]

2. The Decision to Buy Insurance

We assume individuals have the utility function u, which is strictly increasing, strictly concave, at least twice differentiable. Individuals have initial wealth w. They face a potential loss l with probability p. We assume the loss is larger than wealth, $l > w$. Many jurisdictions allow for bankruptcy, so that some amount of the individual's wealth is is protected. If there is no bankruptcy protection, then, since consumption cannot be negative, a loss leaves the individual with final wealth of zero. We let $b \geq 0$ denote the wealth protected by bankruptcy; the most the individual can lose is $w - b$. We assume $u(b)$ is finite. We assume insurance is actuarially fairly priced. Then, if the individual buys insurance, full insurance is optimal (Mossin, 1968). Expected utility with full insurance is $u(w - pl)$.

[3] We should point out that analogous results hold in a health insurance setting with state-dependent utility when the cost of treatment is larger than individuals' willingness to pay for treatment. Stromenger and Wambach (2000) and Posey and Thistle (2017) show that, in this case, individuals may be unwilling to buy insurance and that this may lead to an equilibrium with no trade in insurance. Posey and Thistle show that no trade may be second best and that an active market requires cross-subsidization.

[4] We thank Nathan Hendren for this insight.

If the individual does not buy insurance, their expected utility is $U = (1 - p)u(w) + pu(b)$. If $p > w/l$, then the individual cannot pay for full coverage. If $p = (w - b)/l$, the individual will not buy insurance. Buying insurance leaves the individual with final wealth of b with certainty, while not buying insurance leaves the individual with final wealth of b with less than certainty. This implies there is a critical value $p^* < (w - b)/l$, such that the individual will not buy insurance if $p > p^*$.

The net benefit of buying insurance when the loss probability is p and $p \leq (w - b)/l$ is

$$N(p) = u(w - pl) - (1 - p)u(w) - pu(b). \tag{1}$$

Observe that $N(0) = 0$ and $N((w - b)/l) < 0$. We have

$$N'(p) = -u'(w - pl)l + [u(w) - u(b)] \lesseqgtr 0. \tag{2}$$

and $N''(p) = u''(w - pl)l^2 < 0$. If $N'(0) < 0$, then there is never a benefit to buying insurance and $p^* = 0$.[5] If $N'(0) > 0$ then $N(p) > 0$, at least in some neighborhood of zero, and there is a net benefit to buying insurance if the risk of loss is low enough. Then there is a unique p^* such that $N(p^*) = 0$, where $0 < p^* < (w - b)/l$.

This proves the following result:

Proposition 1: Assume $l > w$. Then there is a p^*, where $0 \leq p^* < (w - b)/l$, such that individuals buy full insurance if $p \leq p^*$ and do not buy insurance if $p > p^*$.

The threshold probability p^* is increasing in initial wealth, decreasing in the amount of bankruptcy protection and decreasing in the amount of the loss.

3. Rothschild-Stiglitz Equilibrium

[5] $N'(0)$ is negative (positive) if $[u(w) - u(b)]/l$, the slope of the line connecting the points $(w - l, u(b))$ and $(w, u(w))$ is less (greater) than $u'(w)$, the slope of the tangent line at $(w, u(w))$. For given preferences there is always a loss large enough that the slope of the line is less than the slope of the tangent, so that $N'(0)$ is negative for larger loss amounts and positive for smaller loss amounts.

We now consider the implication of the decision not to buy insurance for equilibrium in the Rothschild-Stiglitz (1976) framework. We assume the proportion λ of the population are high risk types and the proportion $1 - \lambda$ are low risk types. The loss probabilities are p_H and p_L for high risk and low risks, where $0 < p_L < p_H < 1$. Individuals' types are private information. A contract consists of a premium, α, and a net indemnity, β, paid in the event of a loss. Equivalenty, a contract specifies wealth in the no loss state, $w_N = w - \alpha$, and wealth in the losse state, $w_L = w - l + \beta$. We let $c = (w_N, w_L)$ denote a contract. Firms simultaneously offer contracts, and then individuals decide which, if any, contract they will buy. In equilibrium contracts must break even, satisfy the self-selection constraint $U_H(c_H) \geq U_H(c_L)$ and there must be no other contract which, if offered, would earn positive profits. We let c_H*, c_L* denote the Rothschild-Stiglitz equilibrium contracts. In particular, the low risk contract is at the intersection of the self-selection constraint and the low risk fair odds line. The Rothschild-Stiglitz equilbriuim exists if the proportion of high risk is large enough, $\lambda \geq \lambda'$.[6]

Now consider the no trade condition. Let $\bar{p} = \lambda p_H + (1 - \lambda)p_L$ be the average or pooled loss probability. If $l < w$, as in Hendren's case, then the no trade condition can be written as:

$$(1 - p_L)u'(w)/p_L u'(w - l) \geq (1 - \bar{p})/\bar{p} \quad (3)$$

and

$$(1 - p_H)u'(w)/p_H u'(w - l) \geq (1 - p_H)/p_H. \quad (4)$$

Condition (3) is that the low risks are unwilling to pay the actuarially fair price of pooled coverage. As noted in Hendren (2014), in the canonical two type case with $l < w$, condition (4) can only hold for risk averse individuals if $p_H = 1$. Hendren's analysis for losses less than wealth

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[6] Here we make the assumption used in Rothschild and Stiglitz (1976) that insurers can offer only a single contract. If menus of contracts are allowed, then there exists a $\lambda'' > \lambda'$ such that a menus of contracts cannot break the Rothschild-Stiglitz equilibrium if $\lambda > \lambda''$.

and two risk types with p_H <1 implies that either there is an equilibrium where non-null contracts are traded (if $\lambda > \lambda'$) or equilibrium does not exist (if $\lambda < \lambda'$).[7] We show that if losses are larger than wealth Hendren's no trade condition does not apply.

The Rothschild-Stiglitz equilibrium when losses are larger than wealth is illustrated in Figure 1. If individuals are uninsured, they are at the point $E_0 = (w, b)$. The traditional fair odds lines for high and low risks, P_H and P_L, emanate from $E_1 = (w, w - l)$ since the insurer must receive a premium on all of the coverage provided. Only the portion of the fairs odds lines above the horizontal axis are relvant. For each risk type, the wealth combinations actually faced by the policyholder level off at the horizontal axis, at $w_L = b$, for all w_N to the right of the point of intersection between the traditional fair odds line the horizontal axis. The pooled fair odds line is P_P and levels off in a similar way.

Let λ' be the value of λ at which the fair pooled price line is just tangent to the low risk indifference curve through E_0.

We want to prove the following result:

Proposition 2: Assume $1 > p_H > p^* > \bar{p}$ and $l > w$. If $\lambda > \lambda'$, then the unique Rothschild-Stiglitz equilibrium contracts are the null contracts, $c_H^* = c_L^* = (w, b)$.
If $\lambda \leq \lambda'$, then equilibrium does not exist.

The proof is straightforward. The curve U_H^0 and U_L^0, the high risk and low risk indifference curves through E_0, are the individual rationality constraints. The low risks would prefer to buy any policy along P_L above point B, and would prefer to remain uninsured rather than buy a policy along P_L below B. Since $p_H > p_H^*$, the high risk fair odds line lies completely below the indifference curve U_H^0, so the high risks do not insure. The indifference curve U_H^0 is

[7] See Hendren's (2013) Theorem 1 and the subsequent discussion.

the self-selection constraint. Since U_H^0 is flatter than U_L^0, it must intersect the fair odds line P_L below B, for example, at point A. The low risks prefer to remain uninsured. If the pooled fair odds line, labeled P_P in Figure 1, lies below the indifference curve U_L^0, i.e., if $\lambda > \lambda'$, then the equilibrium is at the null contracts, $c_H^* = c_L^* = (w, b)$. There is no contract offering a positive level of coverage that at least breaks even and statisfies the self-selection constraint and the individual rationality constraints. Therefore, the equilibrium is unique.

Now suppose $\lambda \leq \lambda'$, so the pooled fair odds line P_P is tangent to or intersects the low risk indifference curve U_L^0. Then a firm can offer a pooled policy, say, c_P, on P_P that attracts both high and low risks, breaking the equilibrium at the null contracts. But the low risk indifference curve is steeper than the high risk indifference curve through c_P. Then a firm can offer a profitable contract that attracts low risks but not high risks, breaking the equilibrium at c_P. Therefore, equilibrium does not exist. ||

Since both high and low risks obtain the null contract, this is a pooling equilibrium. However, the more important characteristic of the equilibrium is that no trade occurs. [8]

As in the standard model where losses are less that wealth, the existence of equilibrium depends on the location of the pooled fair odds line. In Figure 1 the pooled fair odds line P_P lies below the low risk indifference curve U_L^0. Since the proportion of high risks is large enough, the equilibrium exists. If the proportion of high risks were too small, then the pooled fair odds line would intersect the low risk indifference curve. Since there cannot be a pooling equilibrium, the Rothschild-Stiglitz equilibrium would fail to exist. Thus, there is either an equilibrium with no trade or there is no equilibrium.

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[8] The result in Proposition 2 can be extended to more than two risk types. Suppose there are n types, with $0 < p_1 < p_2 < \ldots < p_n < 1$ and that $p_n > p^*$. Then applying the proof of Proposition 2 *seriatim*, the equilibrium contract for all types is the null contract.

The insurer must pay the full amount of the loss, l. Bankruptcy protects the individual's wealth up to the limit b at no explicit cost. The individual only needs to insure their wealth above b. Thus, individuals face a type-specific fixed cost of $p_i(l - b)$, with an actuarially fair mariginal price for coverage above b.[9] If $p_H > p^*$, then the high risks regard the fixed charge as too large, even for full insurance. As a result, they do not buy insurance. The low risks, who face a smaller fixed charge, would be willing to buy insurance if it provided enough coverage. The need to separate the high and low risks limits the amount of coverage that can be offered to the low risks. The low risks regard their fixed charge as too large for the limited coverage they are offered and also do not buy insurance. In the standard model where losses are less than wealth, the fixed charges are zero. This suggests why Hendren's no trade conditions, equations (3) and (4), apply in the standard model, but do not apply when losses are larger than wealth.

It is interesting to consider how the equilibrium changes as p_H changes (we assume equilibrium exists).[10] Suppose that $p_H = p^*$. Then the high risk fair odds line would be tangent to the indifference curve U_H^0 at full insurance. The high risks would be fully insured. However, U_H^0 is still the self-selection constraint, and still intersects the low risk fair odds line at point A. Then the equilibrium contracts would be the full insurance contract for the high risks and the null contract for the low risks. For $p_H < p^*$, the high risk contract is the full insurance contract at the intersection of the high risk fair odds line and the 45-degree full insurance line. As p_H falls the high risk contract moves up along the full insurance line. The intersection between the high risk indifference curve that is the self-selection constraint and the low risk fair odds line moves up along the low risk fair odds line. As long as the intersection is below B the low risks will not

[9] This is analogous to a two-part tariff, where the marginal price determines the quantity of the good sold and the fixed fee extracts the consumer surplus (see, e.g., Tirole, 1988, pp. 143-148).

[10] The critical value λ', increases as p_H gets closer to p_L.

insure. There is a probability, $p_H' < p*$, at which the intersection is at B. Then for $p_H < p_H'$, we have the usual Rothschild-Stiglitz equilibrium where the high risks full insure and the low risks receive partial coverage.

4. Wilson-Miyazaki-Spence Equilbrium.

Two important limitations of the Rothschild-Stiglitz equilibrium are the restriction of competition in single contracts rather than menus of contracts and that equilibrium may fail to exist. This leads us to consider the Wilson-Miyaaki-Spence equilibrium. We show that equilibrium always exists. We show that no trade equilibria can occur when firms compete in menus of contracts and that active trade requires that the low risks subsidize the high risks.

The Wilson-Miyazaki-Spence equilibrium is the solution to the constrained maximization problem:

$$\text{Max } U(c_L)$$

Subject to

$$U_H(c_H) \geq U_H(c_L) \qquad (SS_H)$$

$$U_L(c_L) \geq U_L(c_H) \qquad (SS_L)$$

$$\lambda[w - p_H l - (1 - p_H)w_{HN} - p_H w_{HL}]I_H + \qquad \text{(RC)}$$

$$(1 - \lambda)[w - p_L l - (1 - p_L)w_{LN}) + p_L w_{LL})]I_L$$

$$U_H(c_H) \geq U_H(c_H{*}) \qquad (UH)$$

$$U_H(c_H) \geq U_H(0) \qquad (IR_H)$$

$$U_L(c_L) \geq U_L(0) \qquad (IR_L)$$

The first two constraints are the self-selection constraints for the high and low risk, the third constraint is the resource constraint, and the fourth constraint is a bound on high risks' expected utility. The last two constrains are the individual rationality constraint for the high and low risks. The resource constraint is affected by whether or not high risks, low risks or both decide to buy insurance. The I_i are indicators equal to 1 if the constraint IR_i is slack and equal to 0 if IR_i is binding. Also, we have $U_i(c_i) = (1 - p_i)u(w_{iN}) + p_i u(w_{iL})$ if the individual buys insurance, and $U_i(0) = (1 - p_i)u(w) + p_i u(b)$ if the individual does not buy insurance.

Let δ and μ_H denote the Lagrangian multipliers for the constraints *UH* and SS_H.

Proposition 3: Assume $1 > p_H > p^* > \bar{p}$. The WMS constracts satisfy the following conditions:

A. (1) The resource constraint is binding, (2) either both *IR* constraints are binding or both IR constraints are slack and (3) the solution is unique.

B. If IR_H and IR_L are both slack, then (1) the high risks fully insure, (2) SS_H is binding, and (3)

$$\frac{dw_{LL}}{dw_{LN}} = -\frac{(1-p_L)u'(w_{LN})}{p_L u'(w_{LL})} - \frac{\lambda(1-p_H)u'(w_{LN})+(1-\lambda)(1-p_L)u\prime(w_H)(1+\delta/\mu_H)}{\lambda p_H u'(w_{LL})+(1-\lambda)p_L u\prime(w_H)(1+\delta/\mu_H)} \tag{5}$$

C. If both IR_H and IR_L are binding, then (1) both SS_H and SS_L are binding, (2) both types obtain the null contract.

Proof: The proof is given in the Appendix.

Propostion 3 leaves unanswered the question of when the IR costraints are binding and when they are slack. Define the feasible contract curve (FCC) as the locus of low risk contracts that statisfy the resource constraint and the self-selection constraint for a high risk full insurance contract. Let c_P^0 be the pooled full insurance contract. Let c_H^0 be the contract at the intersection of the high risk fair odds line and the full insurance line. Let c_L^0 be the contract at the intersection of the high risk indifference curve $U_H(c_H^0)$ and the low risk fair odds line. The FCC connects c_P^0 and c_L^0; the slope of the FCC is given by the right-hand side of (4.1). This is illustrated in Figure 2 where the FCC is shown as the dashed line. Let λ'' be the largest value of λ such that the FCC is tangent to the low risk IR constraint, $U_L(0)$. Since the FCC lies above the pooled fair odds line, $\lambda'' > \lambda'$. The FCC is Figure 2 is drawn so that $\lambda = \lambda''$, and the FCC is just tangent to the low risk indifference curve at $\tilde{c}_L$. The contracts $\tilde{c}_H$ and $\tilde{c}_L$ satisfy SS_H, $U_H(\tilde{c}_H) = U_H(\tilde{c}_L)$. If $\lambda > \lambda''$ then IR_L is binding, which implies IR_H is binding, and the WMS equilibrium is at the null contracts. If $\lambda \leq \lambda''$, the WMS equilibrium is at the contracts, $(\tilde{c}_H, \tilde{c}_L)$ where the low risk indifference curve is tangent to the FCC and the self-selection constraint SS_H is binding. At this equilibrium, the low risks subsidize the high risks.

> **Corollary**: Assume $1 > p_H > p^* > \bar{p}$. (*a*) If $\lambda > \lambda''$, the WMS equilibrium is at the null contracts. (*b*) If $\lambda \leq \lambda''$, then the WMS equilibrium is at the cross-subsidized contracts $(\tilde{c}_H, \tilde{c}_L)$.

In order for there to be active trade in the market, the low risks must subsidize the high risks.

5. Conclusion.

We show that, if losses are larger than wealth, there is a threshold where individuals will not insure if the loss probability is above the threshold. In an insurance market with adverse selection, if the high risks' loss probability is above the threshold, then no trade occurs at the Rothschild-Stiglitz equilibrium. We extend the analysis to the Wilson-Miyazaki-Spence equilibrium. We show that, if the proportion of high risks is large, then no trade occurs in equilibrium. If the proportion of high risks is small enough, then there is an equilibrium with active trade in insurance in which the high risks are subsidized. Hendren's no trade condition does not apply when losses are larger than wealth. However, Hendren's broader point, that the existence of active trade equilibria in markets with adverse selection is fragile, remains valid.

Our results have implications for public policy and the regulation of insurance markets. Our results suggest that insurance markets may not function well for low income individuals. The no trade equilibrium is second best (Proposition 3) implying intervention in the market will not lead to welfare improvments. But this considers only the welfare of insurers and (potential) insureds. It does not consider the uncompensated losses born by, say, accident victims or health care providers. From a broader social welfare persective, it may be desirable to implement polcies that lead to an active insurance market.

Assuming it is socially desireable to have active trade in the issurance market, there are two main policy alternatives. The first is to reduce the level of wealth protected by bankruptcy. A general reduction in bankruptcy protection will have repercussions beyond the insurance market. A possibility would be to eliminate some forms of debt, such as liability judgements or certain medical expenses, from discharge in bankruptcy. Reducing bankruptcy protection may offer only a partial solution since it may not be sufficient to induce high risk individuals to buy insurance.

The second alternative is to subsidize insurance purchases. When losses are larger then wealth, individuals face a type-specific fixed charge equal to the portion of the loss they would not have to pay scaled by their risk of loss, $p_i(l - b)$. A subsidy to eliminate this fixed charge would lead to active trade in insurance. The subsidies could be implemented without the regulator having knowledge of individuals' risk types. Analgous to Crocker and Snow (1985), policies with a premium to coverage ratio ($\alpha/(\alpha + \beta)$ in our notation) above the average loss probability would receive a subsidy equal to the high risk fixed charge while polices with a premium to coverage ratio below the average loss probability would receive a subsidy equal to the low risk fixed charge. In terms of Figure 1, the subsidy shifts E_1 up to E_0. This requires tax revenue from other sources in order to relax the resource constraint and induce individuals to buy insurance. The expected cost is the same, so the subsidy shifts the cost from those who would bear the uncompensated losses to taxpayers. Whether this is desireable or not is a social value judgement about which group will bear the cost of unsubsidized losses.

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FIGURE 1
Rothschild-Stiglitz Equilibrium with Large Losses

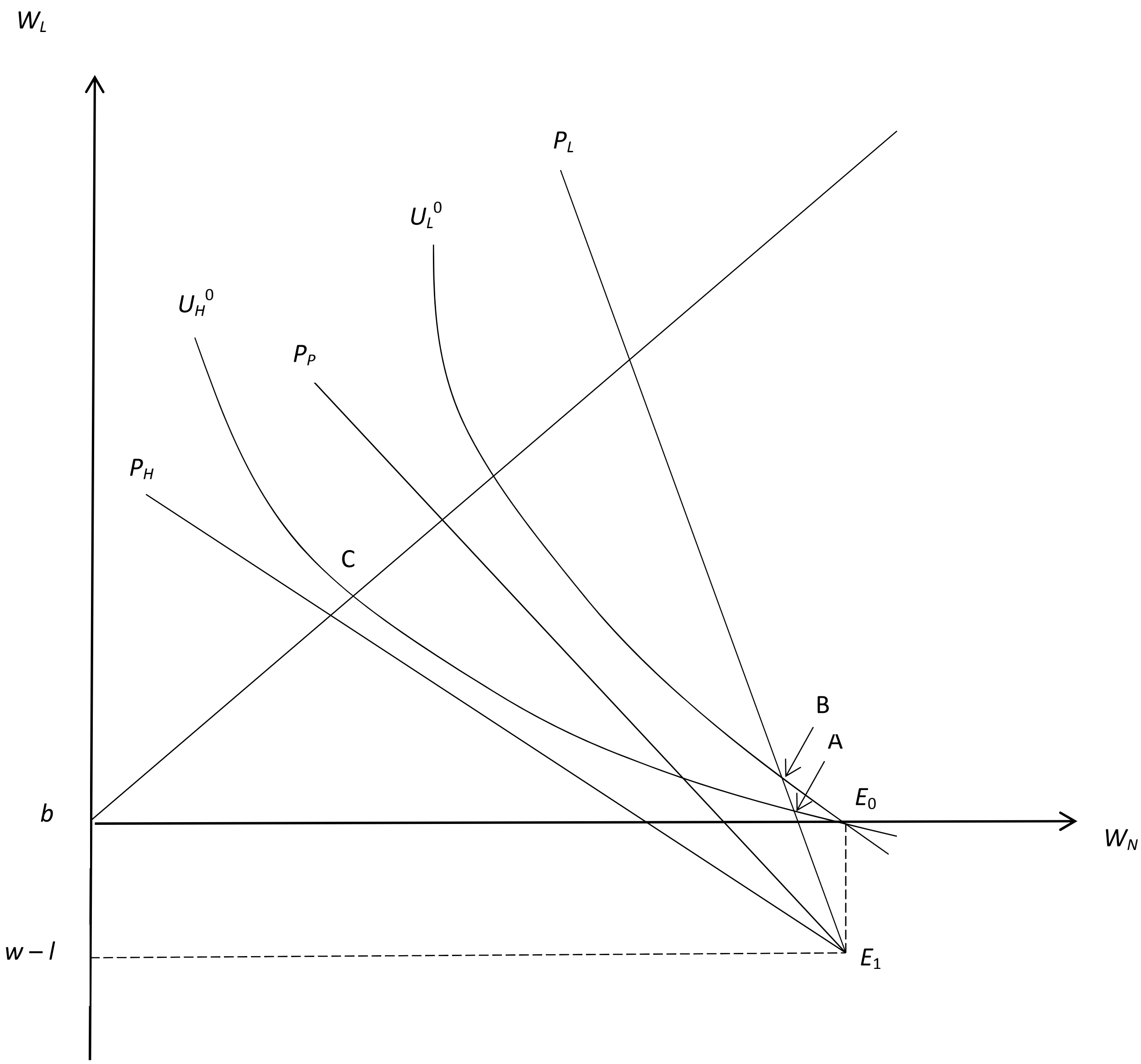
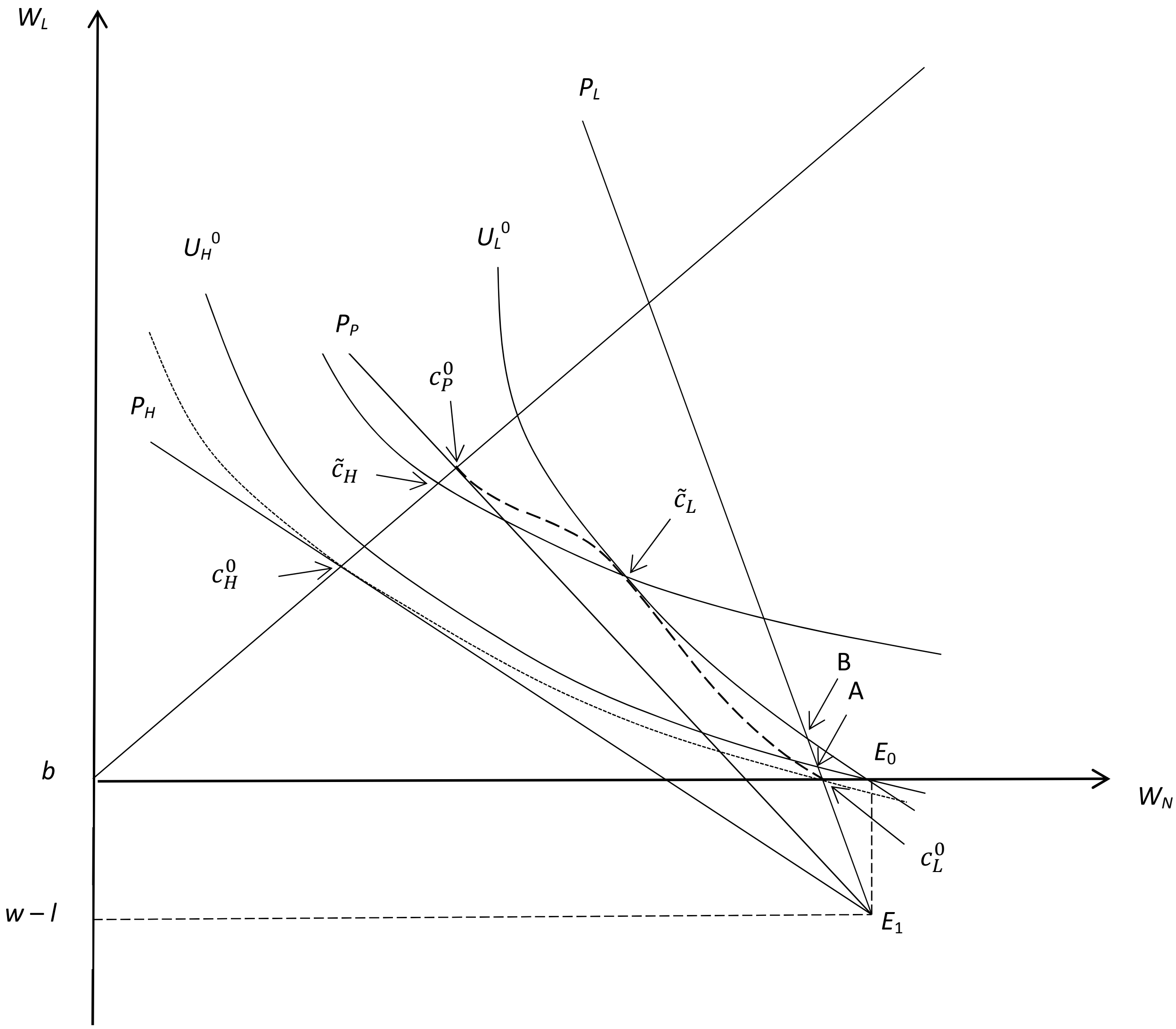


FIGURE 2
WMS Equilibrium with Large Losses



Appendix A: Proof of Proposition 3.

A. (1) If at least one type buys insurance, then $I_i = 1$ for some i, and non-satiation implies the resource constraint is binding. If neither type buys insurance, then $I_H = I_L = 0$ and the resource constraint becomes 0 = 0. (2) We need to show (*i*) IR_H binding implies IR_L binding, (*ii*) IR_L binding implies IR_H binding, (*iii*) IR_H slack implies IR_L slack and (*iv*) IR_L slack implies IR_H slack. Observe that (*i*) and (*iv*) are equivalent and (*ii*) and (*iii*) are equivalent. We first prove (*ii*). Assume IR_L is binding. Suppose, by way of contradiction, that IR_H is slack, $U_H(c_H) > U_H(0)$. Since the H's purchase a policy with positive coverage, either $p_H < p^*$ or the Hs are subsidized. The first possibility is ruled out by hypothesis. Since the Ls do not purchase insurance the Hs cannot be subsidized without violating the resource constraint. We now prove (*vi*), that if IR_L is slack then IR_H is slack. If IR_L is slack, then $U_L(c_L) > U_L(0)$. Assume, by way of contradiction, the IR_H is binding, $U_H(c_H) = U_H(0)$ Since c_L must break even, it lies on the type L fair odds line above (w, b). But this implies $U_H(c_L) > U_H(0)$, which is the desired contradiction. (3) Uniqueness is proved below.

B. (1) Let (c_H, c_L) be contracts that are resource feasible and satisfy self-selection and where c_H offers partial coverage. Then there is a contract c_H' that offers more coverage either at the same profit or, if the high risks are subsidized, at a higher profit. The high risks prefer c_H' to c_H. If the self-selection constraint for the low risk is binding, they prefer c_H' to c_L and if it is slack, they may prefer c_L. In either case, the contracts (c_H, c_L) do not solve the maximization problem. (2) The Lagrangian for the maximization problem is

$$\mathcal{L} = U_L(c_L) + \mu_H[U_H(c_H) - U_H(c_L)] + \mu_L[U_L(c_L) - U_L(c_H)]$$

$$+ \gamma\{\lambda[w - p_H l - (1 - p_H)w_{HN} - p_H w_{HL}]I_H +$$

$$(1-\lambda)[w-p_L l-(1-p_L)w_{LN})+p_L w_{LL})]I_L\}$$

$$+\ \delta[U_H(c_H)-U_H(c_H{}^*)],$$

where μ_H, μ_L, γ and δ are the Lagrangian multipliers. The first order conditions are

$$\partial \mathcal{L}/\partial w_{LN} = (1+\mu_L)\partial U_L(c_L)/\partial w_{LN} - \mu_H\partial U_H(c_L)/\partial \mathrm{w}_{LN} - \gamma(1-\lambda)(1-p_L) = 0 \quad \text{(A.1)}$$

$$\partial \mathcal{L}/\partial w_{LL} = (1+\mu_L)\partial U_L(c_L)/\partial w_{LL} - \mu_H\partial U_H(c_L)/\partial \mathrm{w}_{LL} - \gamma(1-\lambda)p_L = 0 \quad \text{(A.2)}$$

$$\partial \mathcal{L}/\partial w_{HN} = (\delta+\mu_H)\partial U_H(c_H)/\partial w_{HN} - \mu_L\partial U_L(c_H)/\partial \mathrm{w}_{HN} - \gamma\lambda(1-p_H) = 0 \quad \text{(A.3)}$$

$$\partial \mathcal{L}/\partial w_{HL} = (\delta+\mu_H)\partial U_H(c_H)/\partial w_{HL} - \mu_L\partial U_L(c_H)/\partial \mathrm{w}_{HL} - \gamma\lambda p_L) = 0, \quad \text{(A.4)}$$

along with the complementary slackness conditions.

If both self-selection constraints are slack, then the expected utility of both types can be increased. So at least one self-selection constraint must be binding. From (A.3) and (A.4), we have

$$\frac{(1-p_H)u\prime(w_{HN})}{p_H u\prime(w_{HL})} = \frac{\mu_L(1-p_L)u\prime(w_{HN}) + \gamma\lambda(1-p_H)}{\mu_L p_L u\prime(w_{HL}) + \gamma\lambda p_H} \quad \text{(A.5)}$$

Evaluated at any full insurance contract (w_H, w_H), this becomes

$$\frac{1-p_H}{p_H} = \frac{\mu_L(1-p_L)u'(w_H)+\gamma\lambda(1-p_H)}{\mu_L p_L u'(w_H)+\gamma\lambda p_H} \quad \text{(A.6)}$$

This can hold as an equality if and only if $\mu_L = 0$. If both self-selection constraints are binding, then both types must be at the pooled contract c_P. Then high risk utility is $U_H(c_P)$. From (A.1) and (A.2), we have

$$\frac{(1-p_L)u\prime(w_{LN})}{p_L u\prime(w_{LL})} = \frac{\mu_H(1-p_H)u\prime(w_{LN}) + \gamma(1-\lambda)(1-p_L)}{\mu_H p_H u\prime(w_{LL}) + \gamma(1-\lambda)p_L} \tag{A.7}$$

Evaluated at c_P, this becomes

$$\frac{1-p_L}{p_L} = \frac{\mu_H(1-p_H)u'(w_H) + \gamma(1-\lambda)(1-p_L)}{\mu_H p_H u'(w_H) + \gamma(1-\lambda)p_L} \tag{A.8}$$

This holds as an equality if and only if $\mu_H = 0$. Then (A.6) and (A.8) imply that $\mu_H = \mu_L = 0$ at c_P. If $U_H(c_H) < U_H(c_P)$, then we have $\mu_L = 0$, $\mu_H > 0$, so that the low risk self-selection constraint is slack and the high risk self-selection constraint is binding.

Using (A.3) to eliminate the Lagrangian multiplier γ in (A.7) yields

$$\frac{(1-p_L)u\prime(w_{LN})}{p_L u\prime(w_{LL})} = \frac{\mu_H(1-p_H)u'(w_{LN}) + [\lambda(1-p_H]^{-1}(\mu_H+\delta)(1-\lambda)(1-p_L)u\prime(w_H)}{\mu_H p_H u'(w_{LL}) + [\lambda(1-p_H]^{-1}(\mu_H+\delta)(1-\lambda)p_L u\prime(w_H)} \tag{A.9}$$

Rearranging (A.9) yields (4.1) in the text.

Now suppose that there are two distinct solutions to the constrained maximization problem, $(\tilde{c}_H, \tilde{c}_L)$ and $(\hat{c}_H, \hat{c}_L)$. Then we have $U_L(\tilde{c}_L) = U_L(\hat{c}_L)$. Since the PEC is downward sloping, one of the low risk contracts, say, $\bar{c}_L$, must have more coverage than the other low risk contract. But then the tax on the low risks must be higher, and, since net transfers must balance, the subsidy to the high risks must also be higher. This implies $\overline{w}_H > \widehat{w}_H$ making the high risks are better off. Then the solution $(\tilde{c}_H, \tilde{c}_L)$ Pareto dominates $(\hat{c}_H, \hat{c}_L)$. Therefore $(\tilde{c}_H, \tilde{c}_L)$ is the unique solution.

C. This is Proposition 2. Observe that the IR and SS constraints coincide, so the SS constraints are binding and that the null contract is unique.